

Curs 4

2025/2026

Dispozitive și circuite de microunde pentru radiocomunicații

Disciplina 2025/2026

- 2C/1L (+1), **DCMR (CDM)**
- Minim 7 prezente (curs+laborator)
- Curs - **conf. Radu Damian**
 - Vineri 10-12/**Video (istoric)**, C1, corp C
 - E – **50%** din nota
 - probleme + (2p prez. curs) + (3 teste) + (bonus activitate)
 - primul test L1: ~01.10.2025 (t2 si t3 neanuntate la **curs**)
 - 3pz (C) $\approx$ +0.5p (**2p** max)
 - toate materialele permise

Disciplina 2025/2026

- 2C/1L, **DCMR (CDM)**
- Laborator – **conf. Radu Damian**
 - L – **25%** din nota
 - ADS, 4 sedinte aplicatii
 - prezenta + **rezultate personale!**
 - P – **25%** din nota
 - ADS, 3 sedinte aplicatii (-1? 21-22.12.2022)
 - tema personala

Documentatie

■ <https://rf-opto.etti.tuiasi.ro>



Microwave and Optoelectronics Laboratory
Faculty of Electronics, Telecommunications and Information Technology
Gheorghe Asachi Technical University of Iasi

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[English](#)[Romana](#)

Microwave Devices and Circuits for Radiocommunicati

Course: DCMR (2024-2025)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: DOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, **Sem.** 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Exam**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

[Aggregate Results](#)

Documentatie

- RF-OPTO
 - <https://rf-opto.etti.tuiasi.ro> ~~Moodle~~
- Fotografie
 - de trimis pe [rf-opto](#)
 - necesara la laborator/curs
 - bonus activitate 0.5p/1p (**C4/C6**)

Bibliografie

- <https://rf-opto.etti.tuiasi.ro>
- Irinel Casian-Botez: "Microunde vol. 1: Proiectarea de circuit", Ed. TEHNOPRES, 2008
- David Pozar, Microwave Engineering, Wiley; 4th edition , 2011, ISBN : 978-1-118-29813-8 (E), ISBN : 978-0-470-63155-3 (P)

Examen: Reprezentare logaritmică

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

$$0 \text{ dB} = 1$$

$$+ 0.1 \text{ dB} = 1.023 (+2.3\%)$$

$$+ 3 \text{ dB} = 2$$

$$+ 5 \text{ dB} = 3$$

$$+ 10 \text{ dB} = 10$$

$$-3 \text{ dB} = 0.5$$

$$-10 \text{ dB} = 0.1$$

$$-20 \text{ dB} = 0.01$$

$$-30 \text{ dB} = 0.001$$

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

$$0 \text{ dBm} = 1 \text{ mW}$$

$$3 \text{ dBm} = 2 \text{ mW}$$

$$5 \text{ dBm} = 3 \text{ mW}$$

$$10 \text{ dBm} = 10 \text{ mW}$$

$$20 \text{ dBm} = 100 \text{ mW}$$

$$-3 \text{ dBm} = 0.5 \text{ mW}$$

$$-10 \text{ dBm} = 100 \mu\text{W}$$

$$-30 \text{ dBm} = 1 \mu\text{W}$$

$$-60 \text{ dBm} = 1 \text{ nW}$$

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

$$[x] + [\text{dB}] = [x]$$

Examen

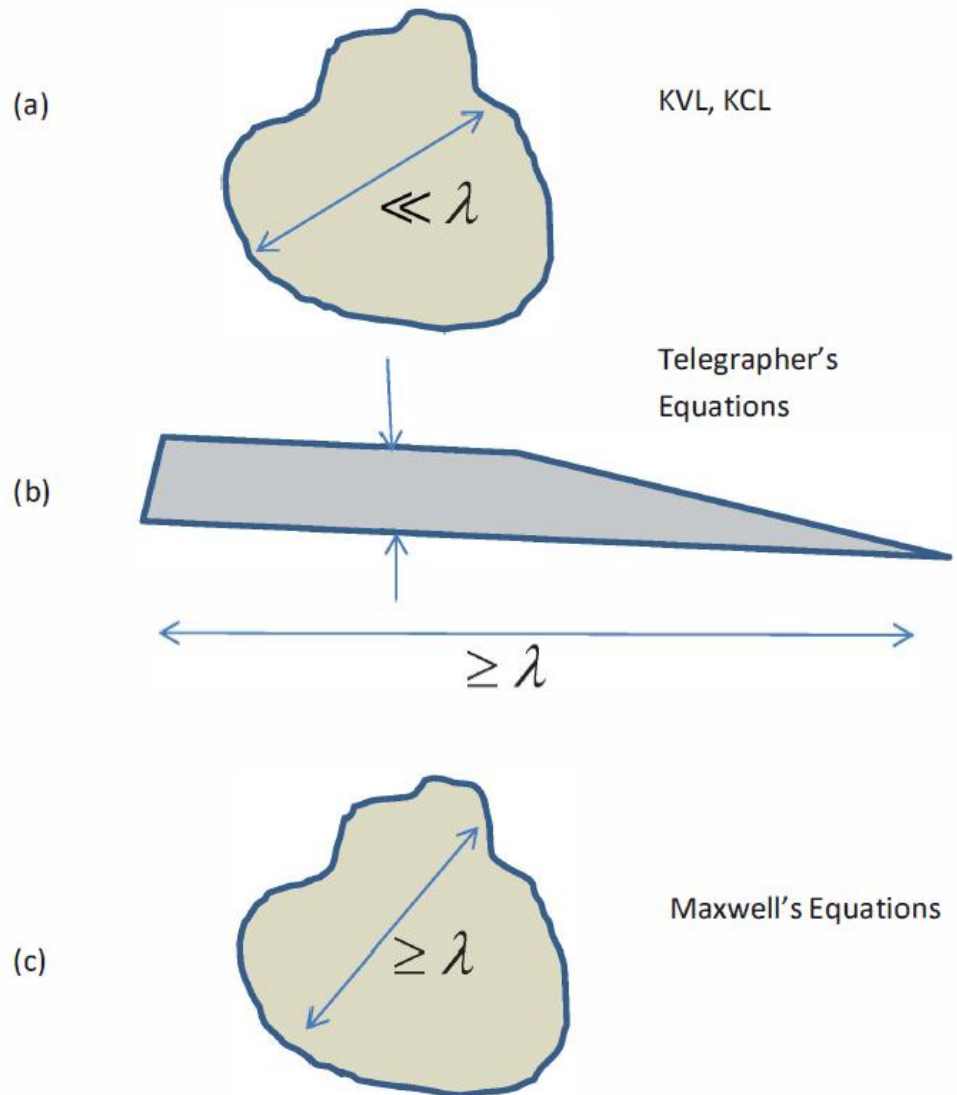
- Operatii cu numere complexe!
- $z = a + j \cdot b ; j^2 = -1$

Lungime electrica

- Comportarea (descrierea) unui circuit depinde de lungimea sa electrica la frecventele de interes

- $E \approx 0 \rightarrow$ Kirchhoff
- $E > 0 \rightarrow$ Maxwell

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



Linii de transmisie in mod TEM

Cuprins

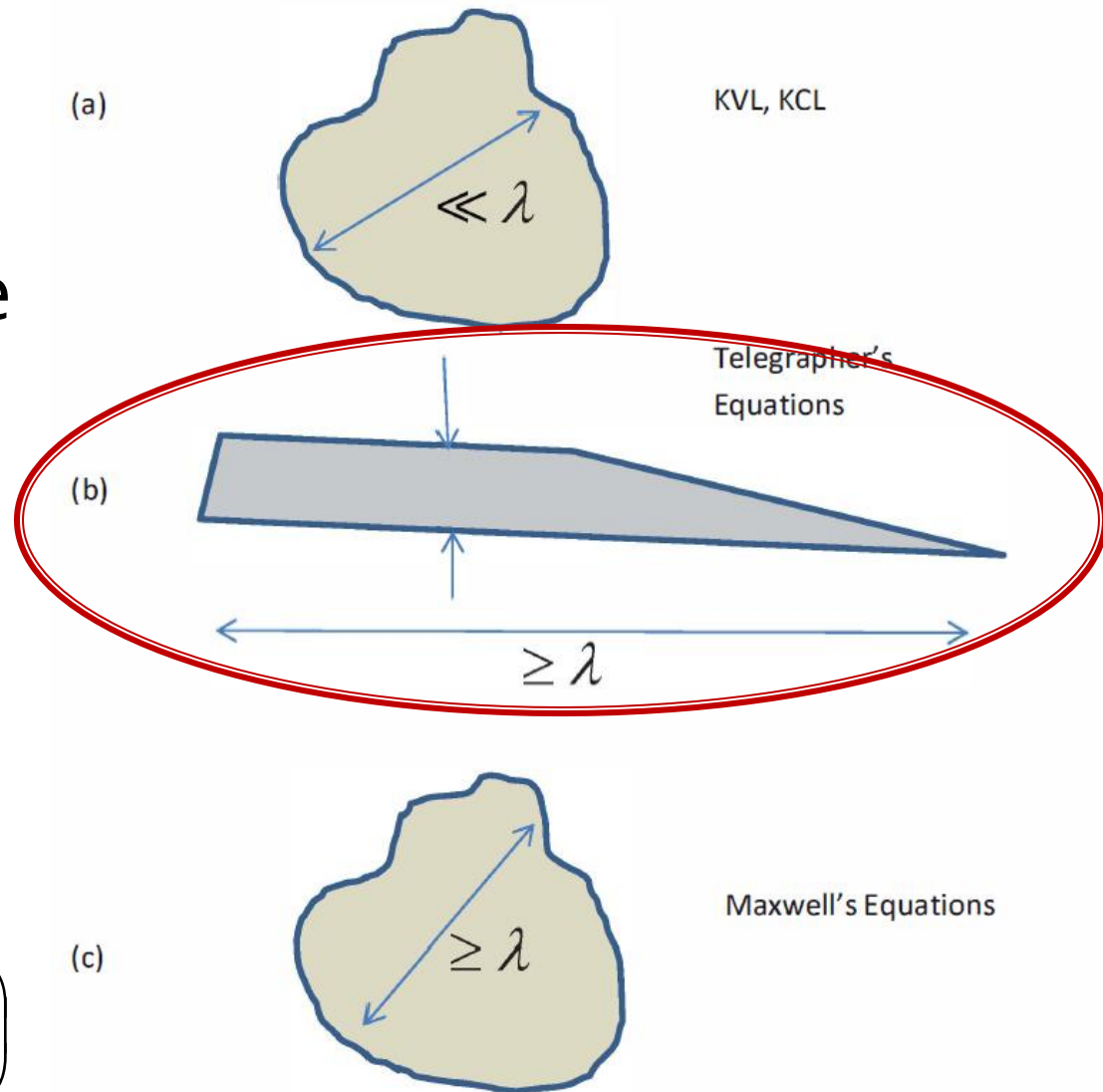
- **Linii de transmisie**
- **Adaptarea de impedanță**
- **Cuploare direcționale**
- **Divizoare de putere**
- **Amplificatoare de microunde**
- **Filtre de microunde**
- **Oscilatoare de microunde ?**

Lungime electrica

- Comportarea (descrierea) unui circuit depinde de lungimea sa electrica la frecventele de interes

- $E \approx 0 \rightarrow$ Kirchhoff
- $E > 0 \rightarrow$ propagare

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



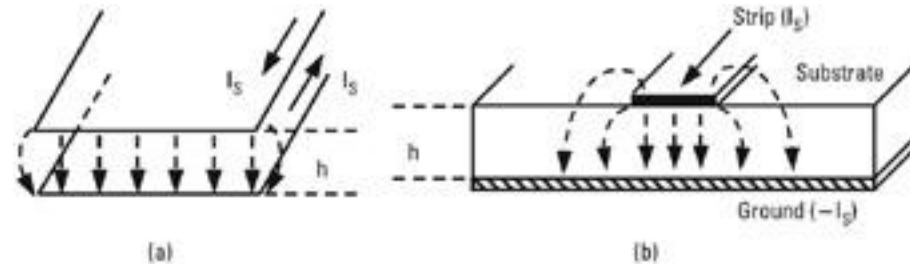
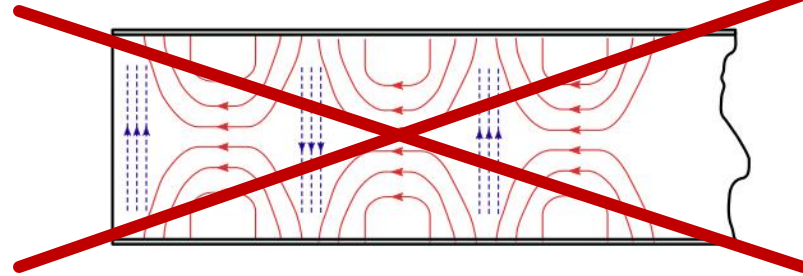
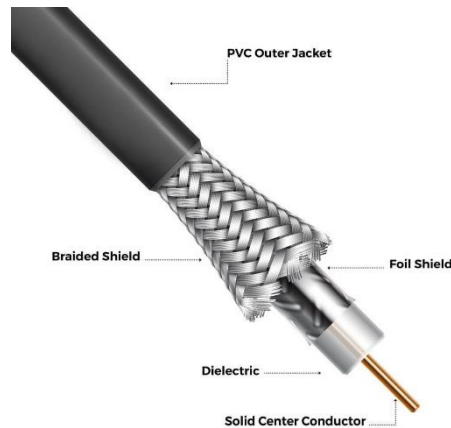
Linie de transmisie

- mod TEM, doi conductori

$I(z,t)$



$V(z,t)$

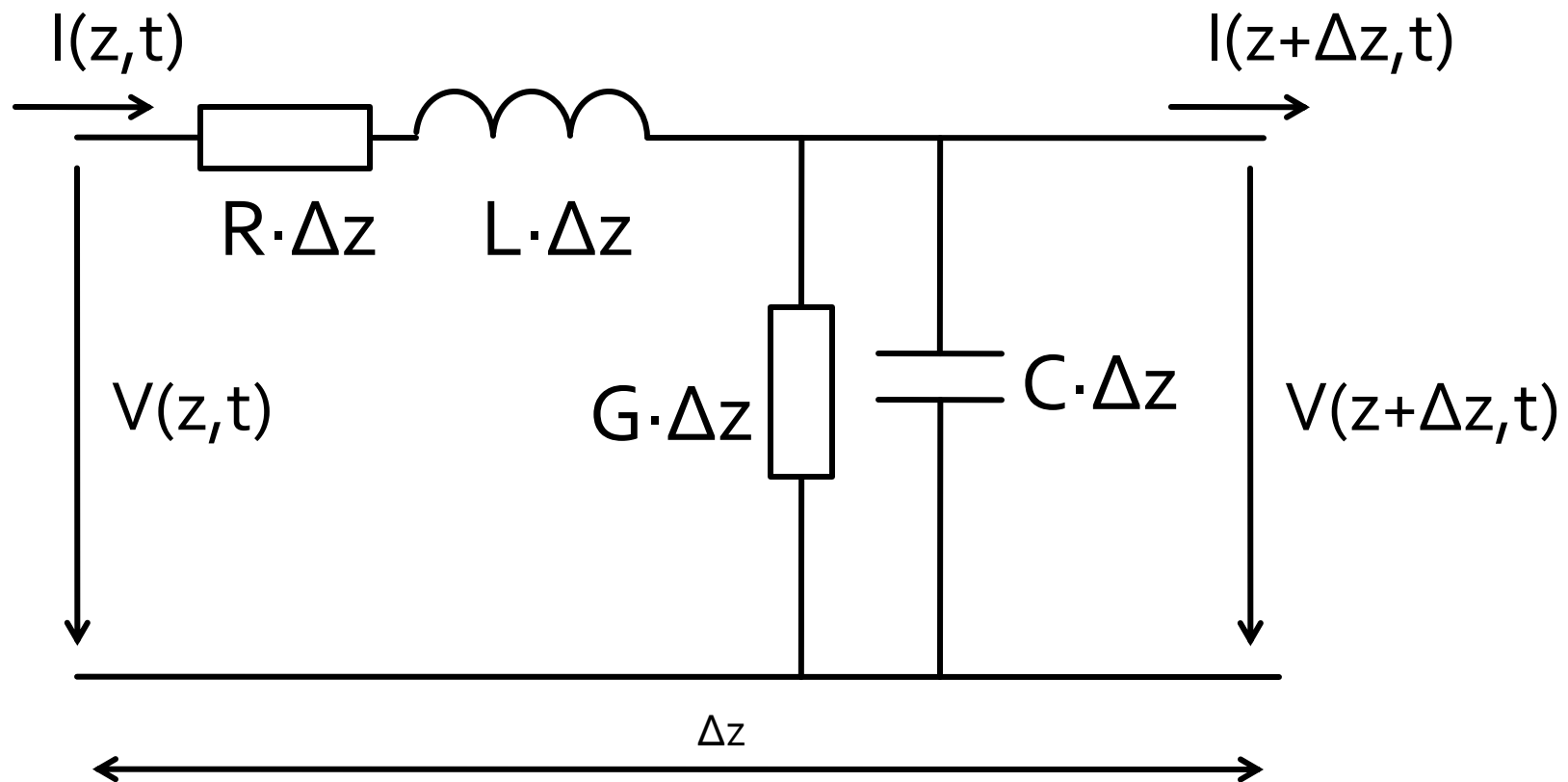


z



Linie de transmisie model echivalent

- mod TEM, doi conductori, parametri **lineici**



- parametri **lineici, distribuiti** R, L, G, C ($\Omega/\text{m}, \text{H}/\text{m} \dots$)

Linie fara pierderi

- **Fara pierderi:** $R=G=0$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)} = j \cdot \omega \cdot \sqrt{L \cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

$$Z_0 = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_0 **real**

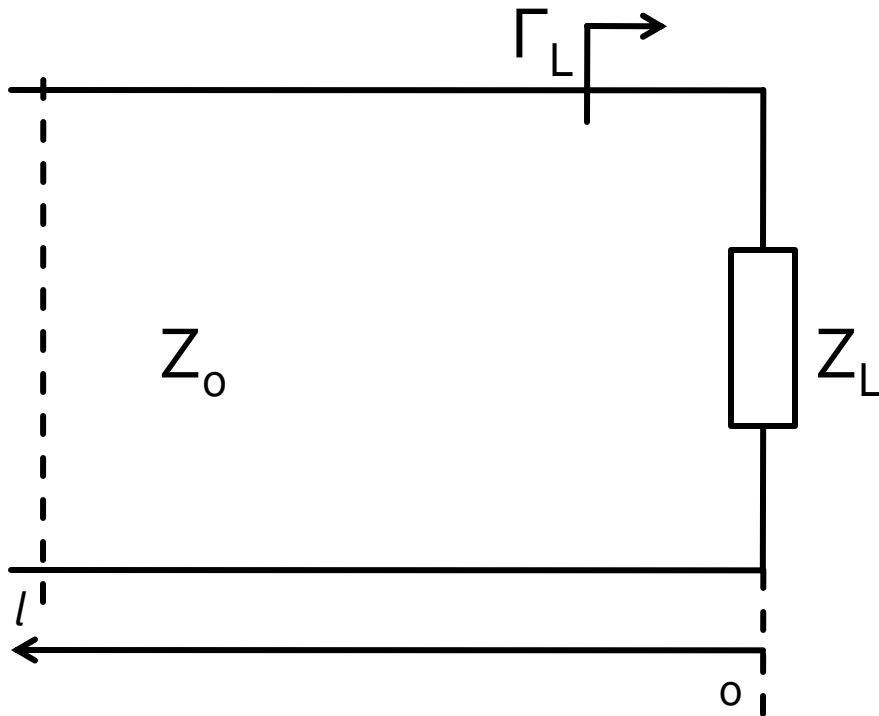
$$\left\{ \begin{array}{l} V(z) = V_0^+ e^{-j \cdot \beta \cdot z} + V_0^- e^{j \cdot \beta \cdot z} \\ I(z) = \frac{V_0^+}{Z_0} e^{-j \cdot \beta \cdot z} - \frac{V_0^-}{Z_0} e^{j \cdot \beta \cdot z} \end{array} \right.$$

$$\lambda = \frac{2\pi}{\omega \cdot \sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

$$e^{-\gamma \cdot z} + e^{\gamma \cdot z} \Rightarrow e^{-j \cdot \beta \cdot z} + e^{j \cdot \beta \cdot z}$$

Linie fara pierderi



$$V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- coeficient de reflexie in tensiune

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_0 real

Linie fara pierderi

$$V(z) = V_0^+ \cdot (e^{-j\beta \cdot z} + \Gamma \cdot e^{j\beta \cdot z})$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta \cdot z} - \Gamma \cdot e^{j\beta \cdot z})$$

■ Puterea medie

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\{1 - \Gamma^* \cdot e^{-2j\beta \cdot z} + \Gamma \cdot e^{2j\beta \cdot z} - |\Gamma|^2\}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

$$(z - z^*) = \text{Im}$$

- Puterea transmisa sarcinii = Puterea incidenta - Puterea "reflectata"

- Return Loss [dB]

$$\text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

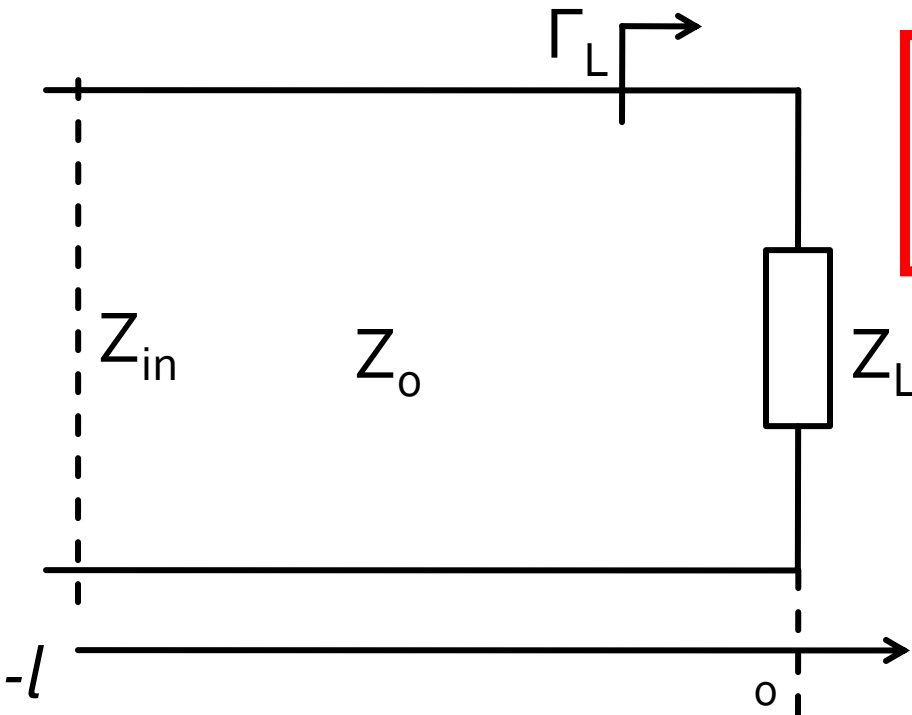
Linie fara pierderi

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

- Puterea medie e constanta in lungul liniei
 - (**fara** $P_{avg}(\mathbf{z})$)
 - poate fi masurata
- Se utilizeaza puterea pentru a caracteriza amplitudinea semnalului
 - un punct de vedere “energetic” (fizica)
 - energie mai mare = semnal “mai mare”

Linie fara pierderi

- impedanta la intrarea liniei de impedanta caracteristica Z_0 , de lungime l , terminata cu impedanta Z_L



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

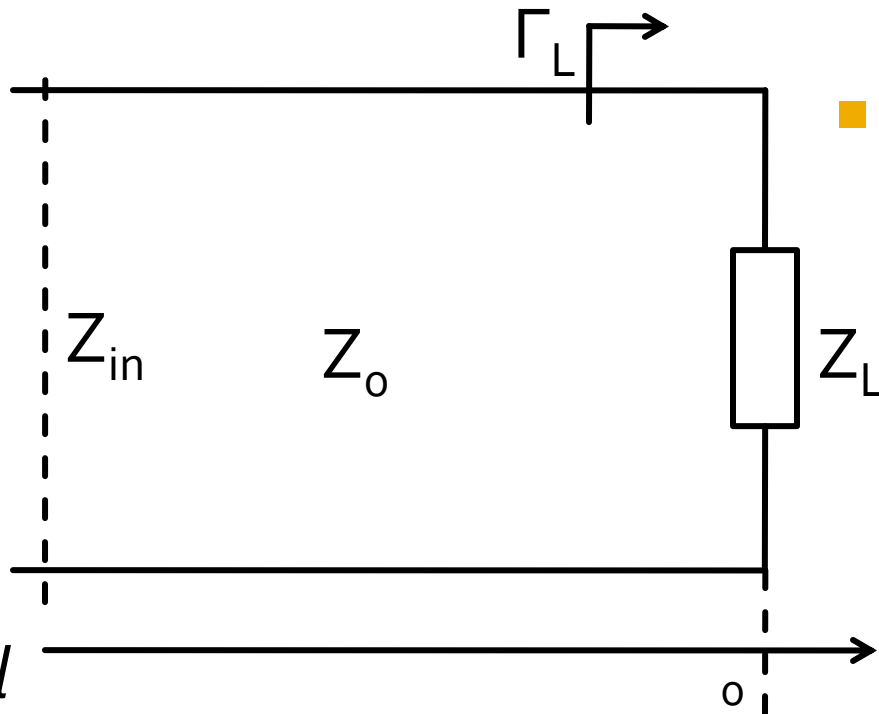
Linie fara pierderi, cazuri particulare

- $l = k \cdot \lambda / 2 \quad \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi \quad \tan \beta \cdot l = 0$

$$Z_{in} = Z_L$$

- $l = \lambda / 4 + k \cdot \lambda / 2 \quad \tan \beta \cdot l \rightarrow \infty$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$



- Transformatorul in sfert de lungime de unda

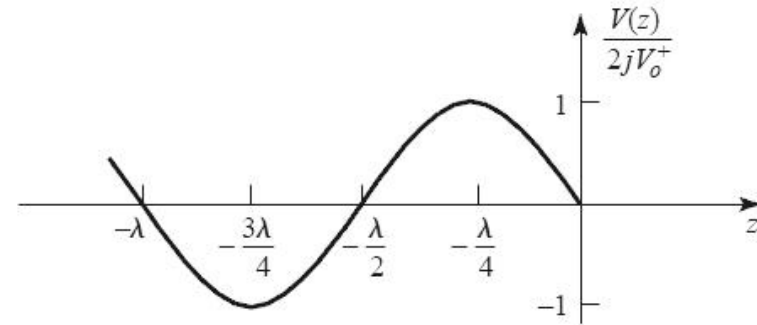
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Linie în scurtcircuit

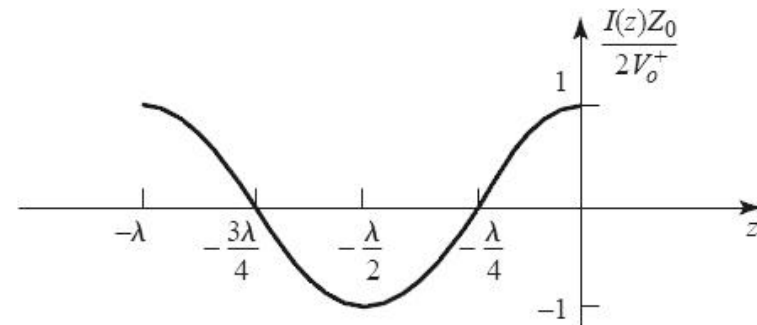
- $Z_L = 0$
- reactanța pură
 - +/- → în funcție de l

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

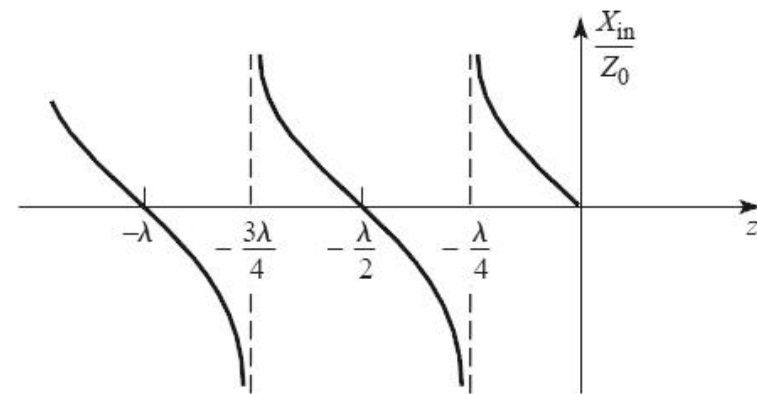
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



(a)



(b)



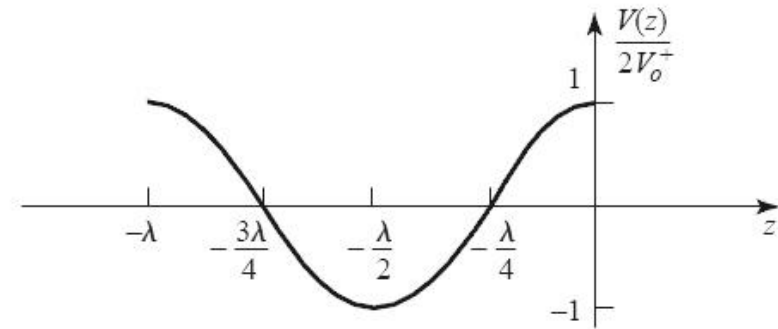
(c)

Linie în gol

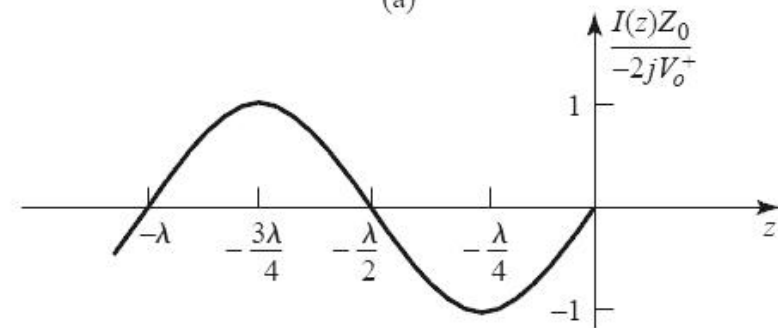
- $Z_L = \infty \rightarrow 1/Z_L = 0$
- reactanța pură
 - +/- $\rightarrow$ în funcție de l

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

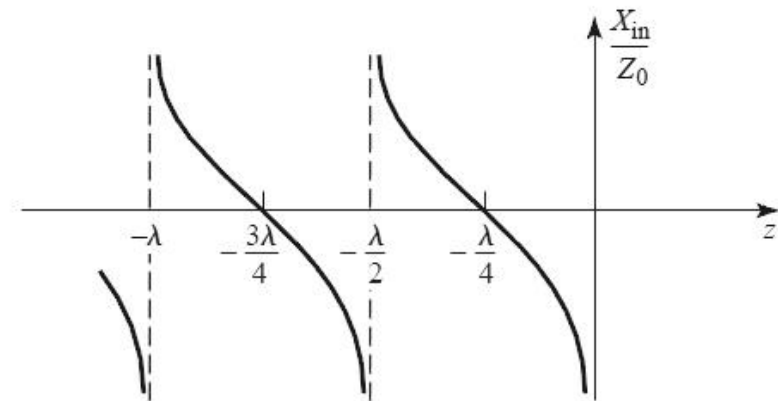
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



(a)

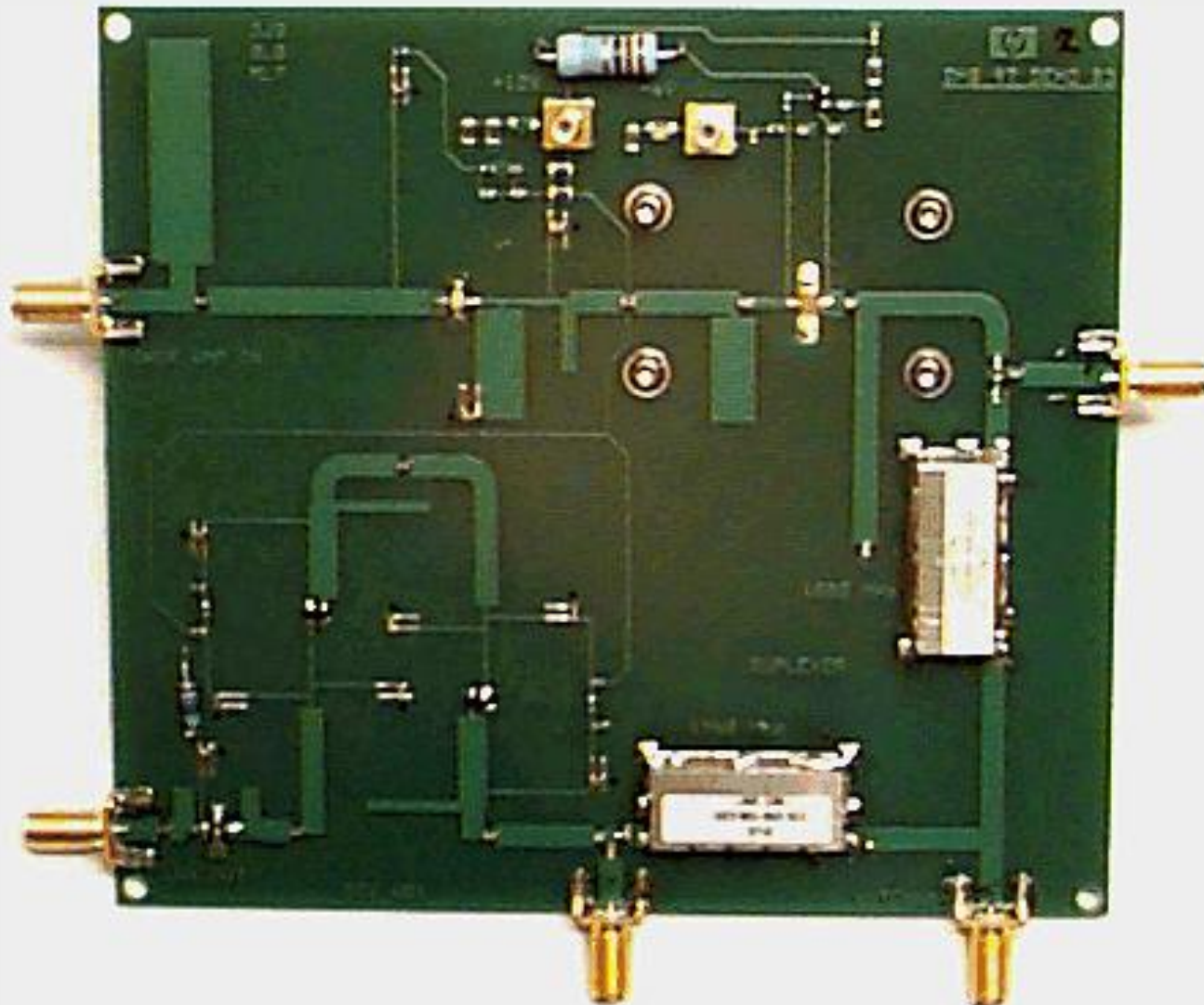


(b)



(c)

Example



Transfer de putere

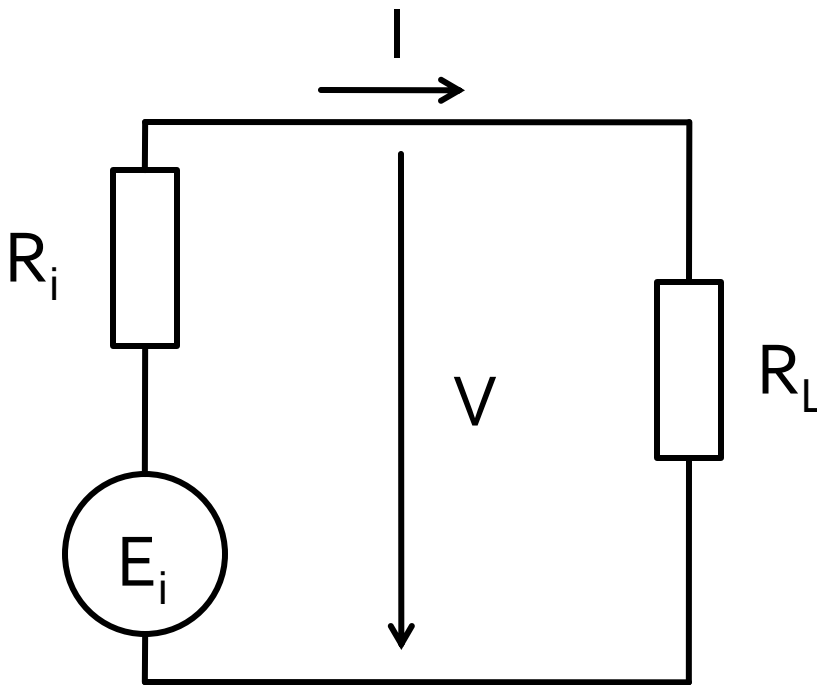
Adaptarea de impedanță

Cuprins

- Linii de transmisie
- **Adaptarea de impedanță**
- Cuploare direcționale
- Divizoare de putere
- Amplificatoare de microunde
- Filtre de microunde
- Oscilatoare de microunde ?

Adaptare, impedante reale

- Generator adaptat la sarcina



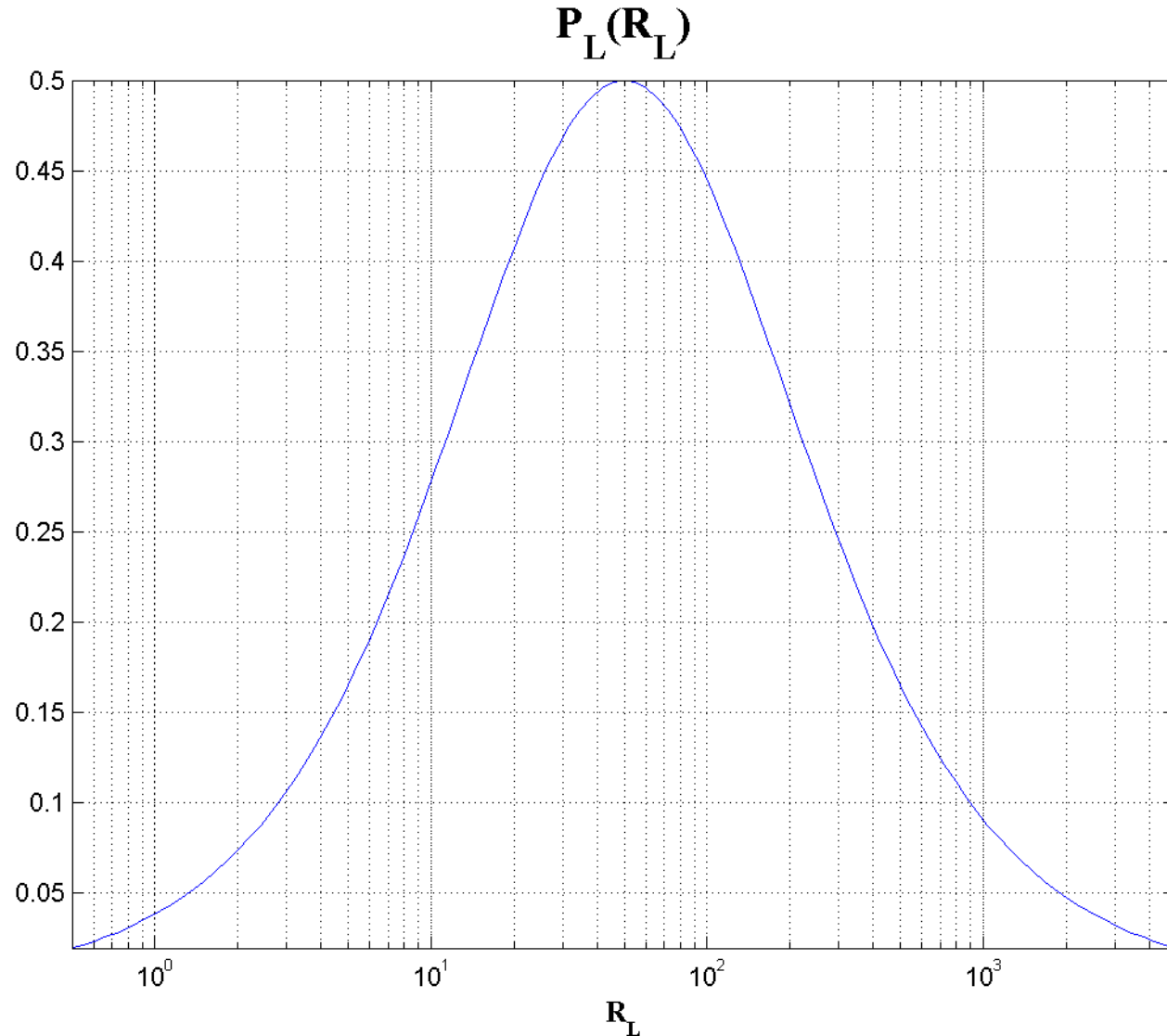
$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

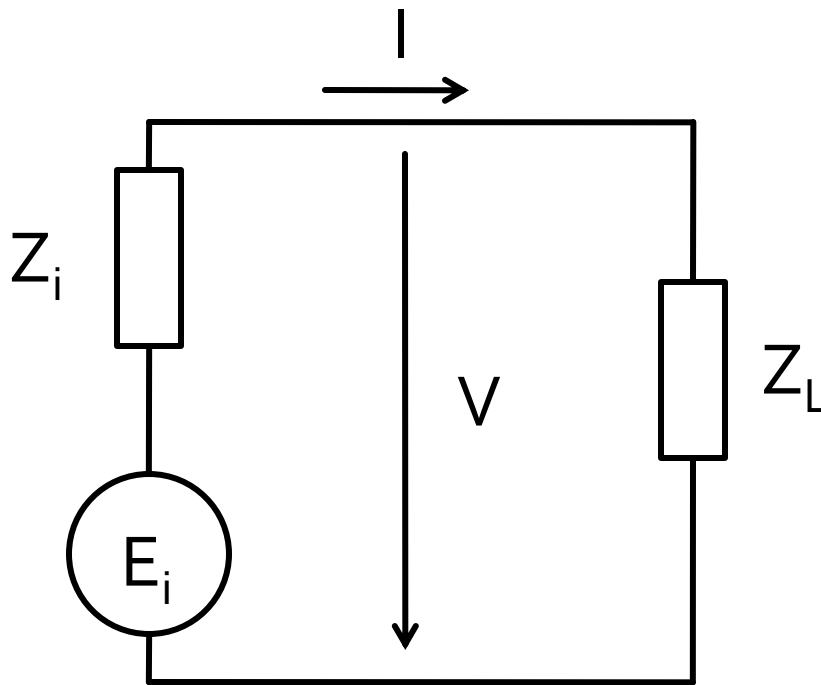
Adaptare , impedante reale



Continuare

Adaptare, impedante complexe

- Generator adaptat la sarcina



$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \operatorname{Re} Z_L \cdot |I|^2$$

$$P_L = \operatorname{Re} Z_L \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

Adaptare

$$P_L = \frac{R_L \cdot |E_i|^2}{|Z_i + Z_L|^2} = \frac{R_L \cdot |E_i|^2}{|(R_i + R_L) + j \cdot (X_i + X_L)|^2}$$

$$|a + j \cdot b| = \sqrt{a^2 + b^2}$$

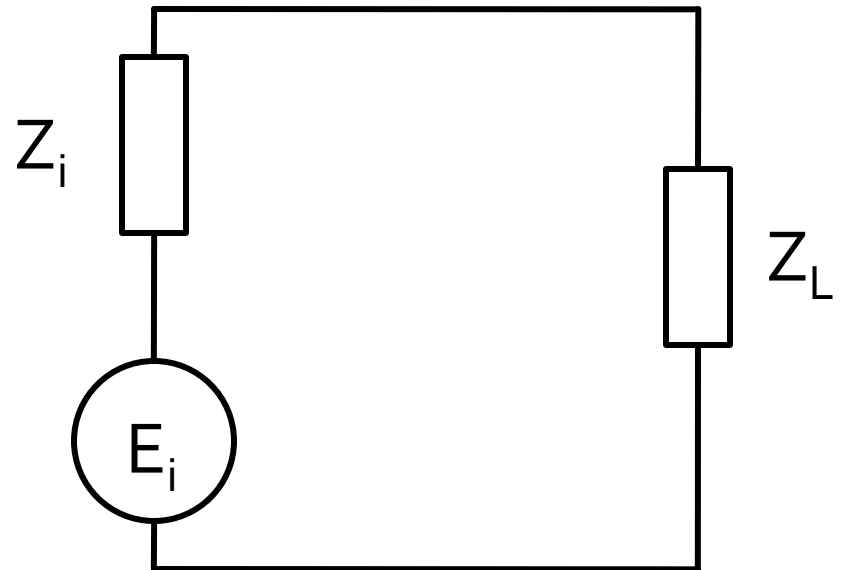
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

- Adaptare
 - putere maxima transmisa sarcinii
 - conditie?

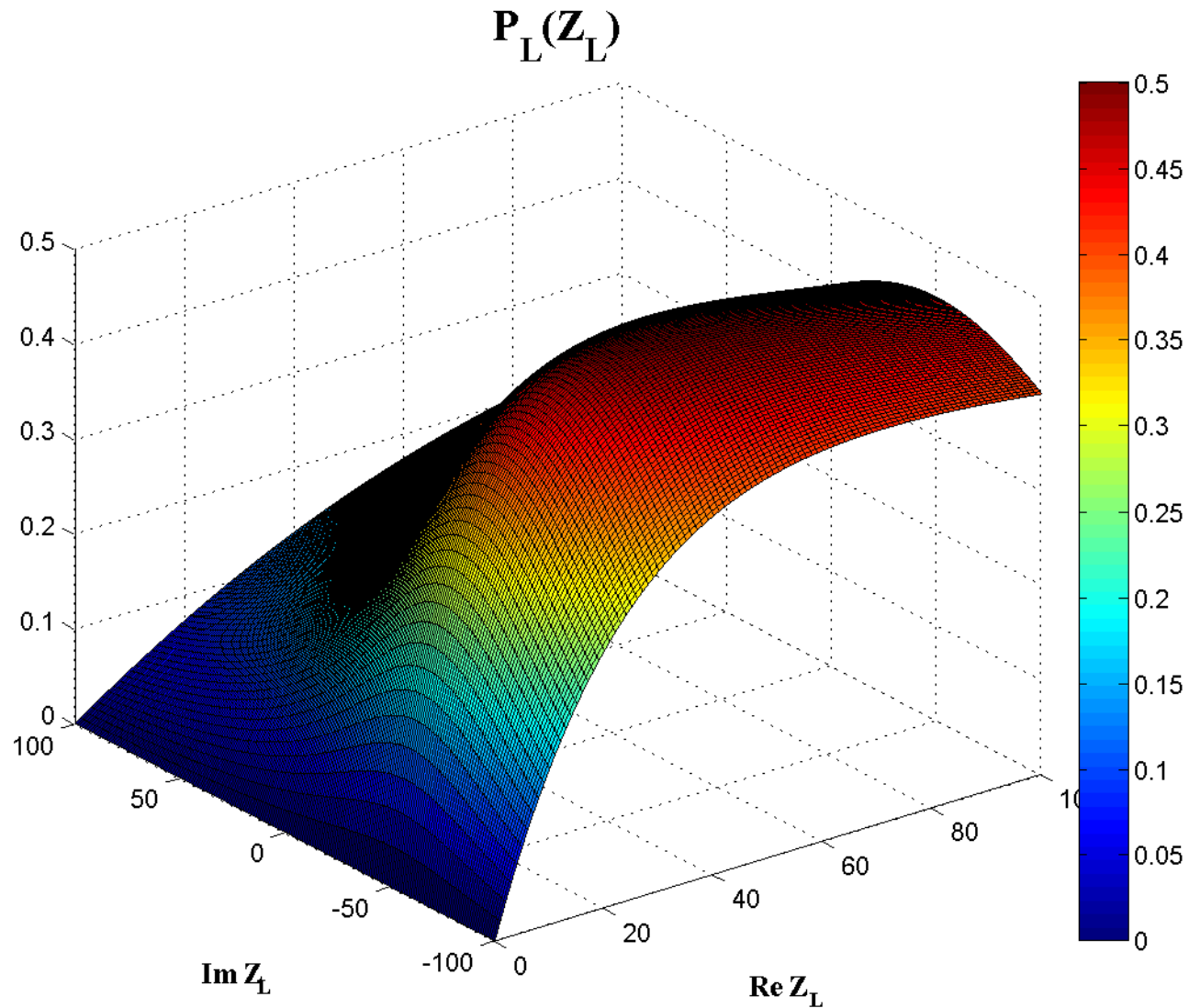
Adaptare, exemplu

- $E = 10\text{V}$
- $Z_i = 50\ \Omega + j \cdot 50\ \Omega$
- $P_L(Z_L)$?

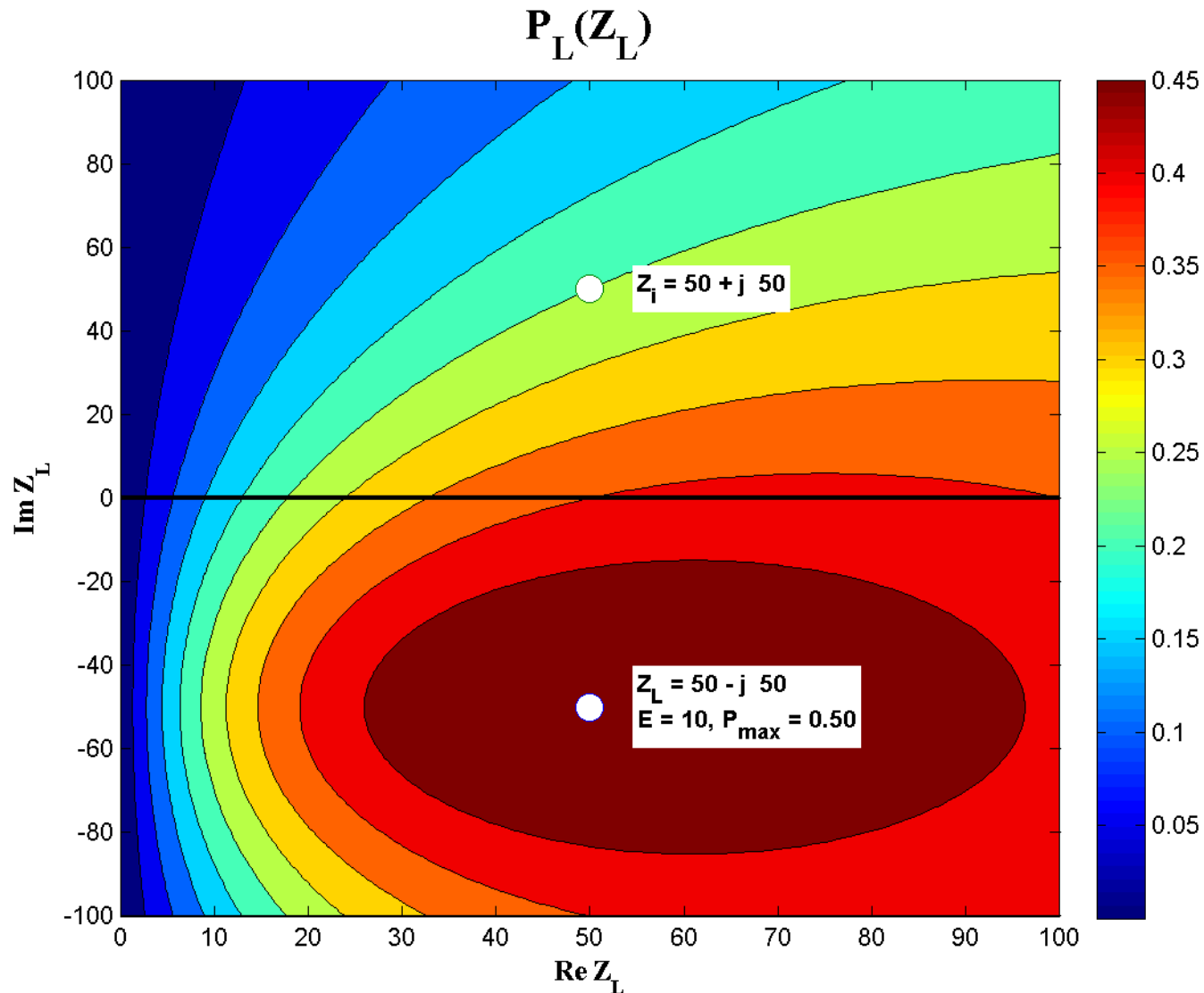
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$



Adaptare, exemplu



Adaptare, exemplu



Adaptare dpdv al puterii

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$R_i > 0, R_L > 0$$

$$P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a$$

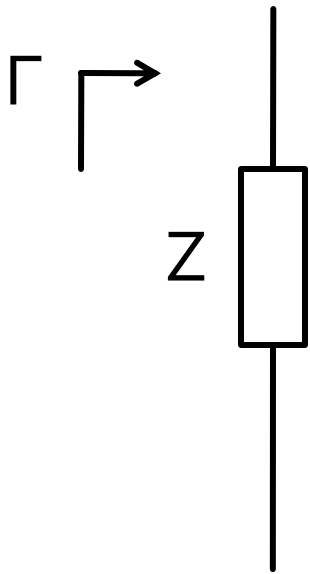
$$R_L = R_i, X_L = -X_i$$

- P_a : Puterea disponibilă (available)

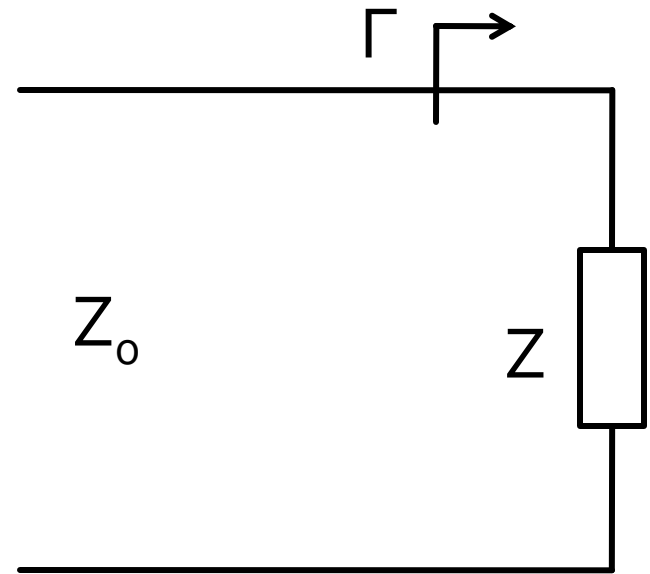
$$Z_L = Z_i^*$$

Coeficient de reflexie

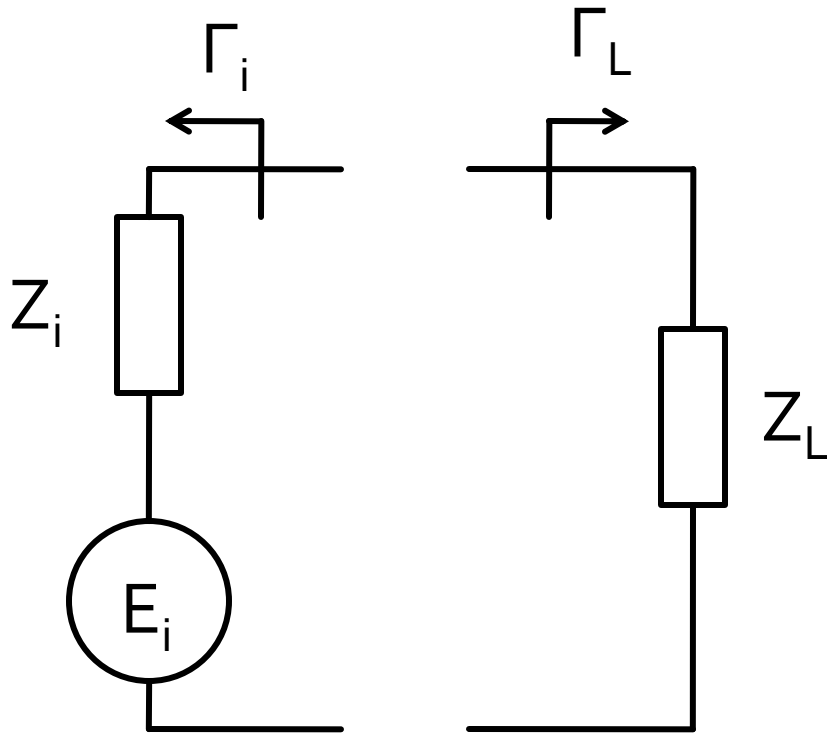
- Un Z_0 oarecare ales ca referinta



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



Adaptare dpdv al puterii



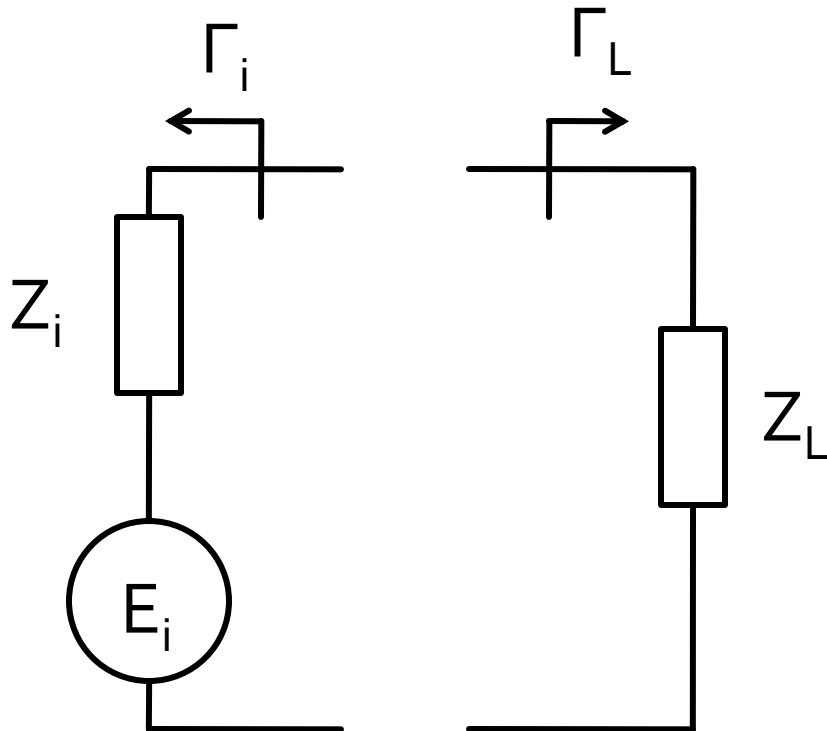
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

Adaptare dpdv al puterii



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

Adaptare: $Z_L = Z_i^*$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0^*} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

Daca se alege un Z_0 real $\Gamma_L = \Gamma_i^*$

Adaptare dpdv al puterii

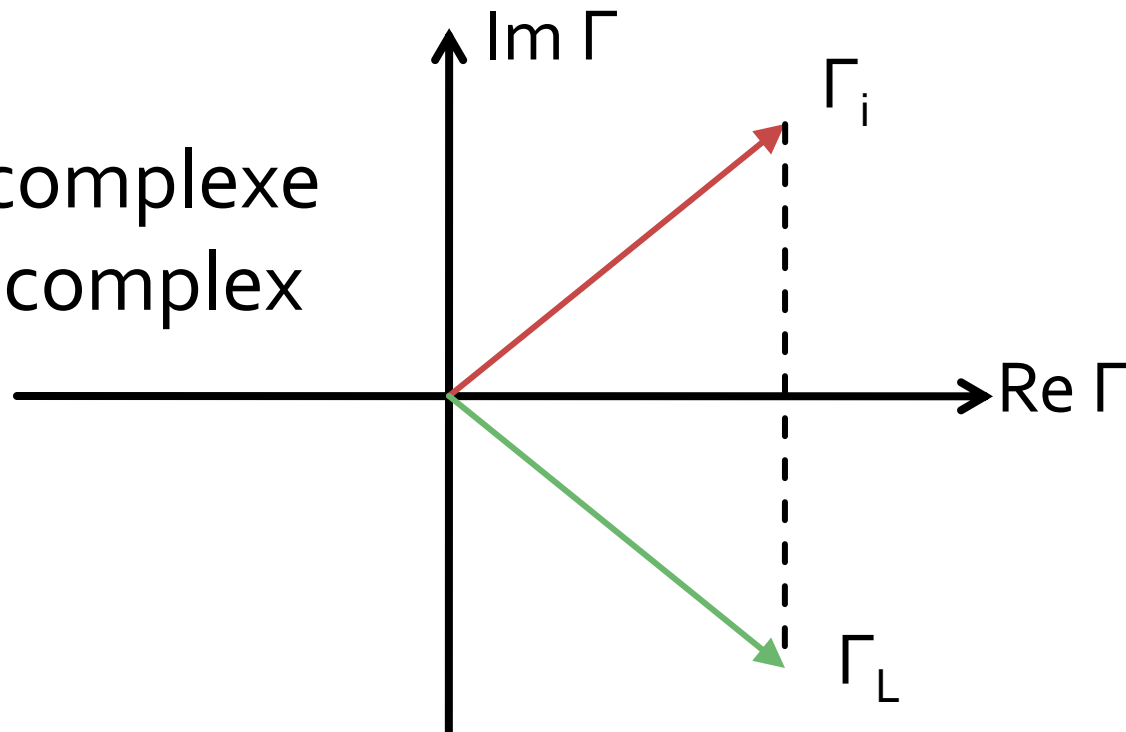
Daca se alege un Z_0 real

$$Z_L = Z_i^*$$

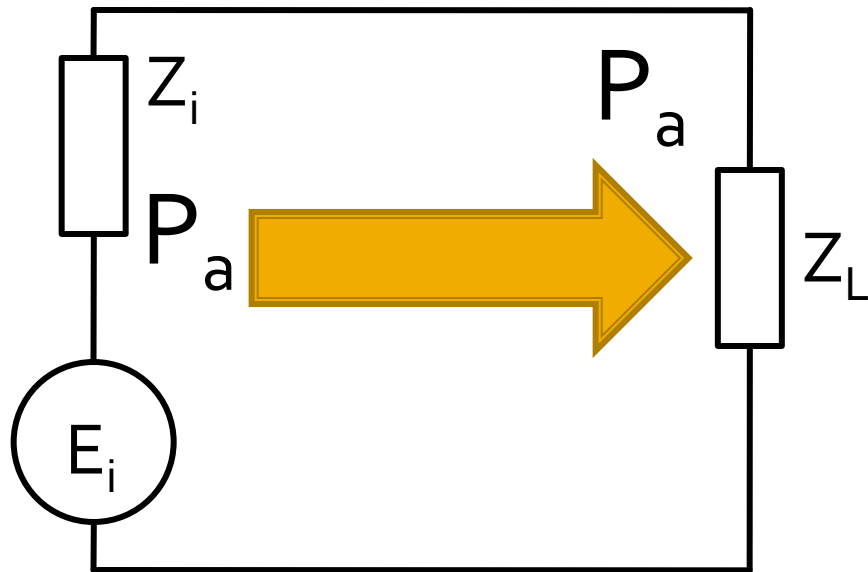
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- numere complexe
- in planul complex

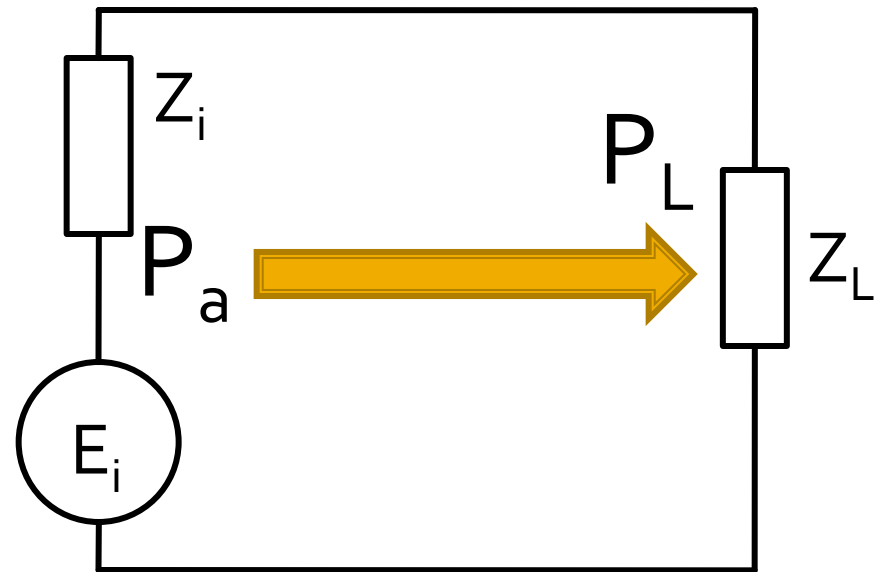


Reflexie de putere / Model



$$Z_L = Z_i^*$$

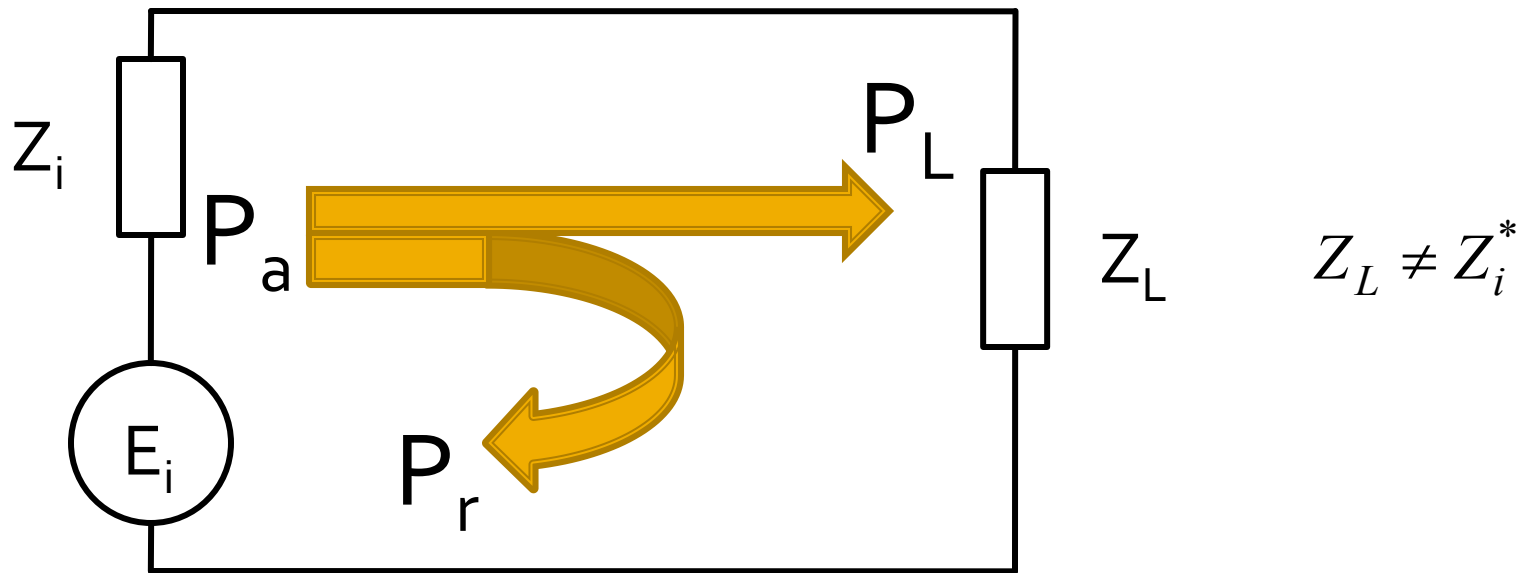
$$P_L = P_a$$



$$Z_L \neq Z_i^*$$

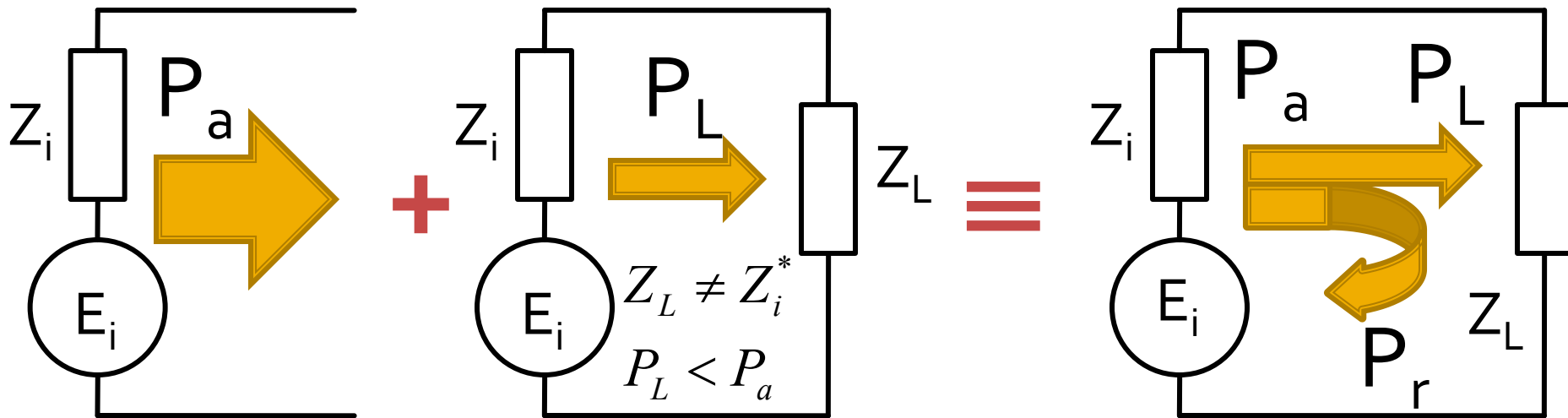
$$P_L < P_a$$

Reflexie de putere / Model



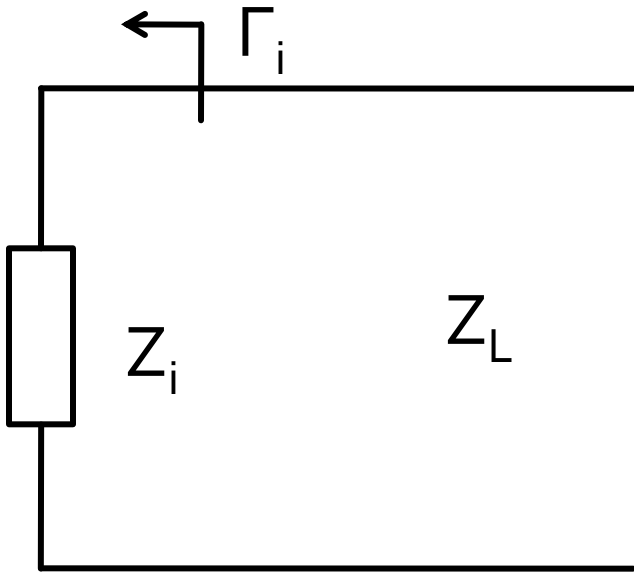
- ~~Putere reflectata~~
- Putere a undei reflectate

Reflexie de putere / Model

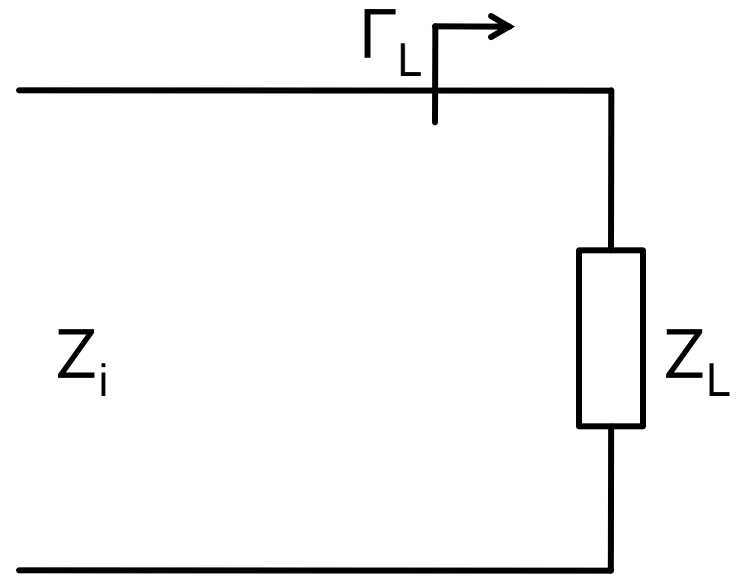


- Generatorul are posibilitatea de a oferi o anumita putere maxima de semnal P_a
- Pentru o sarcina oarecare, acestuia i se ofera o putere de semnal mai mica $P_L < P_a$
- Se intampla **"ca si cum"** (model) o parte din putere se reflecta $P_r = P_a - P_L$
- Puterea este o marime **scalara!**

Coeficienti de reflexie



$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

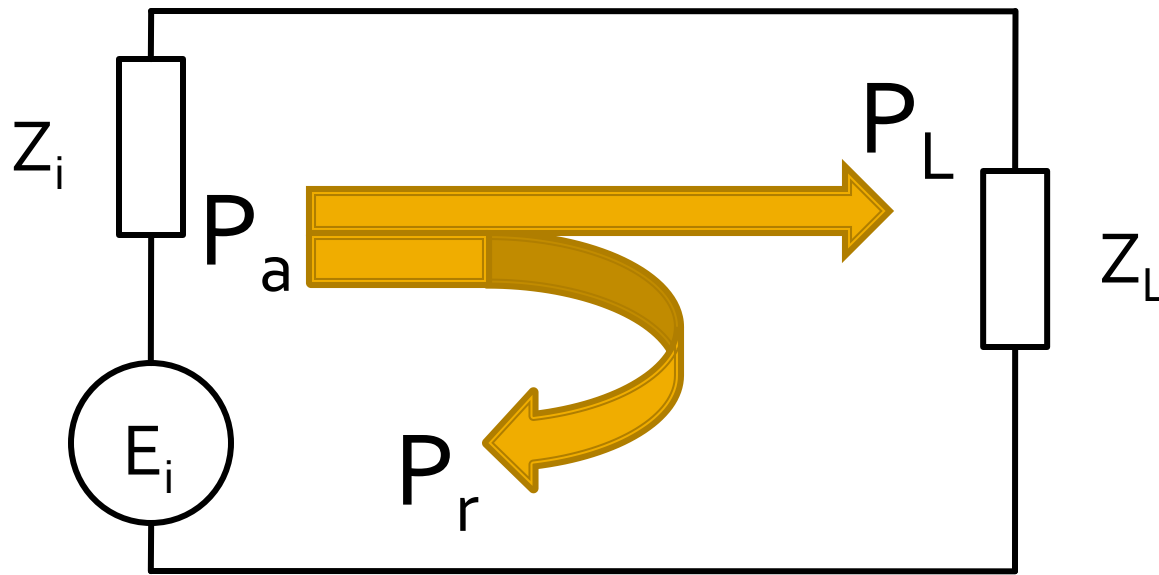
Coeficienti de reflexie

$$\Gamma_i = \frac{(R_i - R_L) + j \cdot (X_i + X_L)}{(R_i + R_L) + j \cdot (X_i + X_L)} \quad \Gamma_L = \frac{(R_L - R_i) + j \cdot (X_L + X_i)}{(R_L + R_i) + j \cdot (X_L + X_i)}$$

$$|\Gamma_i| = \frac{|(R_i - R_L) + j \cdot (X_i + X_L)|}{|(R_i + R_L) + j \cdot (X_i + X_L)|} = \frac{\sqrt{(R_i - R_L)^2 + (X_i + X_L)^2}}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} = |\Gamma_L|$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

Reflexie de putere / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} - \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2} = \frac{|E_i|^2}{4R_i} \cdot \left[1 - \frac{4R_L \cdot R_i}{(R_i + R_L)^2 + (X_i + X_L)^2} \right]$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

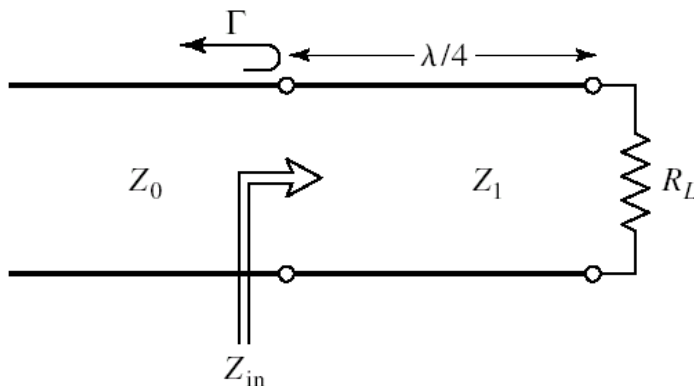
- $|\Gamma|^2$ **este** un coeficient de reflexie in putere

Transformatorul in sfert de lungime de unda

Adaptarea de impedanță

Transformatorul in sfert de lungime de unda

- Feed line – linie de intrare cu impedanta caracteristica Z_0
- Sarcina cu impedanta (**rezistiva!**) R_L
- Dorim adaptarea sarcinei la fider cu o linie de lungime $\lambda/4$ si impedanta caracteristica Z_1

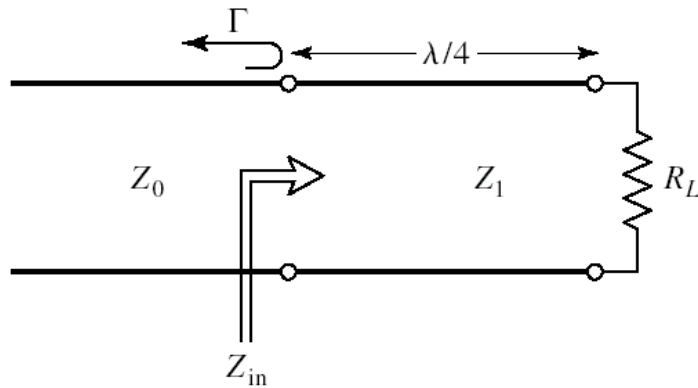


$$Z_{in} = Z_1 \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}}$$

$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

Transformatorul in sfert de lungime de unda



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L}$$

$$\Gamma_{in} = 0 \Rightarrow Z_1 = \sqrt{Z_0 R_L}$$

- Pe fider (Z_0) avem doar unda progresiva
- Pe linia in sfert de lungime de unda (Z_1) avem unda stationara

Transformatorul in sfert de lungime de unda

■ Punct de vedere fizic

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \dots$$

$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

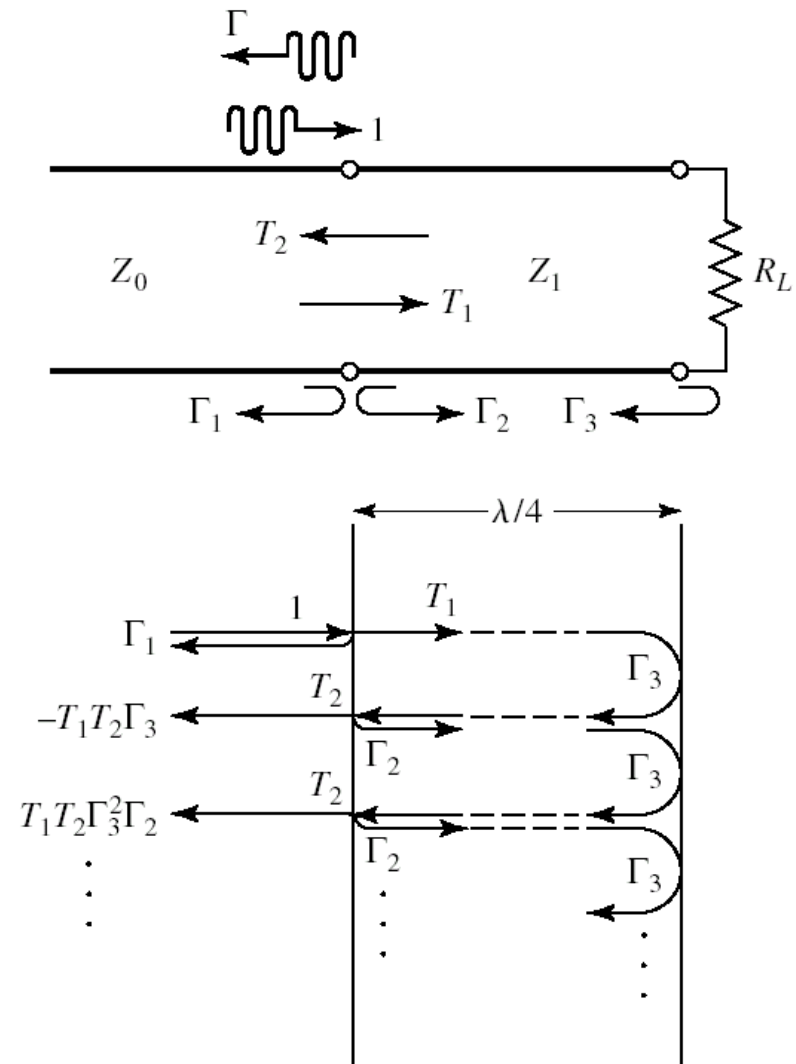
$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$

$$\left. \begin{array}{l} T_1 = \frac{2Z_1}{Z_1 + Z_0} \\ T_2 = \frac{2Z_0}{Z_1 + Z_0} \end{array} \right\} T = 1 - \Gamma$$



Transformatorul in sfert de lungime de unda

■ Punct de vedere fizic

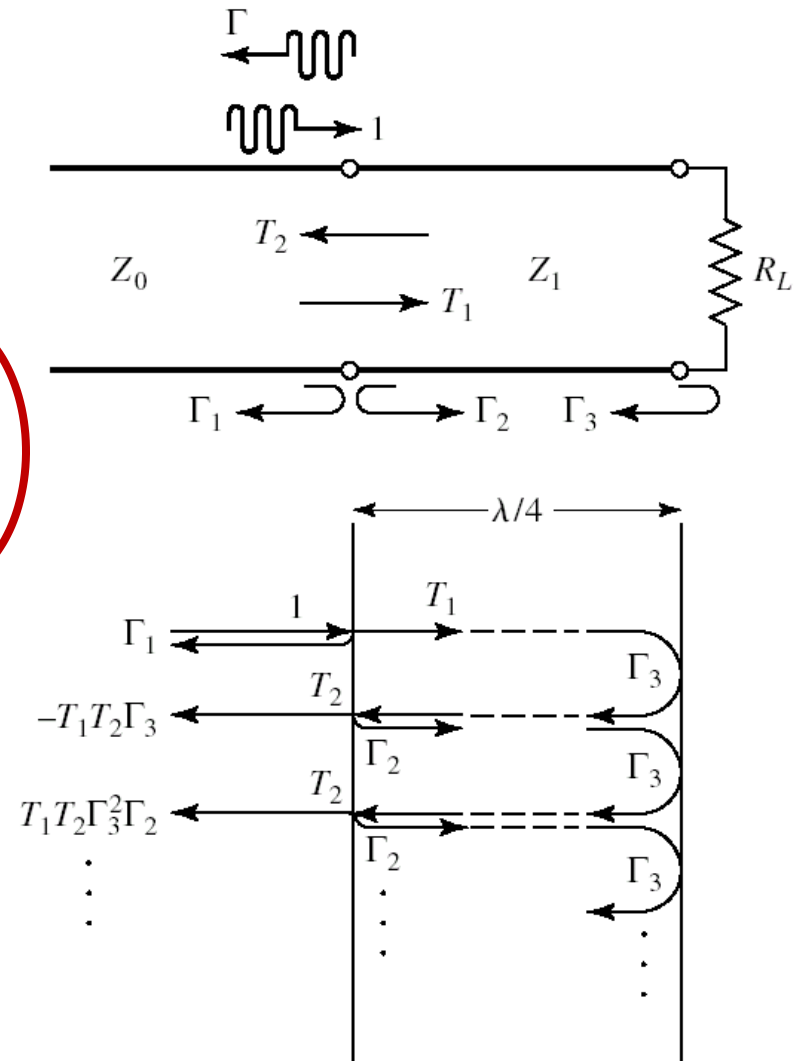
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad \text{for } |x| < 1,$$

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$

$$\Gamma_1 - \Gamma_3(\Gamma_1^2 + T_1 T_2) = \frac{2(Z_1^2 - Z_0 R_L)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma_1^2 + T_1 T_2 = \frac{(Z_1 - Z_0)^2}{(Z_1 + Z_0)^2} + \frac{4Z_1 Z_0}{(Z_1 + Z_0)^2} = 1$$

$$\Gamma = 0 \Leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$



Caracteristica de frecventa

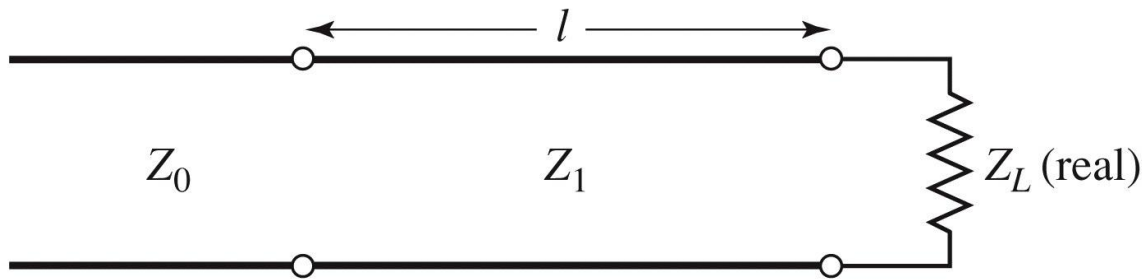


Figure 5.10
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$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

■ **(doar)** la frecventa f_0

$$l = \frac{\lambda_0}{4} \quad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$\theta = \beta \cdot l \quad t = \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt(Z_1^2 - Z_0 Z_L)}{Z_1(Z_L + Z_0) + jt(Z_1^2 + Z_0 Z_L)}$$

$$Z_1^2 = Z_0 \cdot Z_L$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}$$

Caracteristica de frecventa

- calitatea adaptarii $\equiv$ coeficient de reflexie in putere

$$\begin{aligned} |\Gamma| &= \frac{|Z_L - Z_0|}{[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L]^{1/2}} \\ &= \frac{1}{\{(Z_L + Z_0)^2 / (Z_L - Z_0)^2 + [4t^2 Z_0 Z_L / (Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L / (Z_L - Z_0)^2] + [4Z_0 Z_L t^2 / (Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L / (Z_L - Z_0)^2] \sec^2 \theta\}^{1/2}}, \end{aligned}$$

$$\sec \theta = \frac{1}{\cos \theta} \rightarrow$$

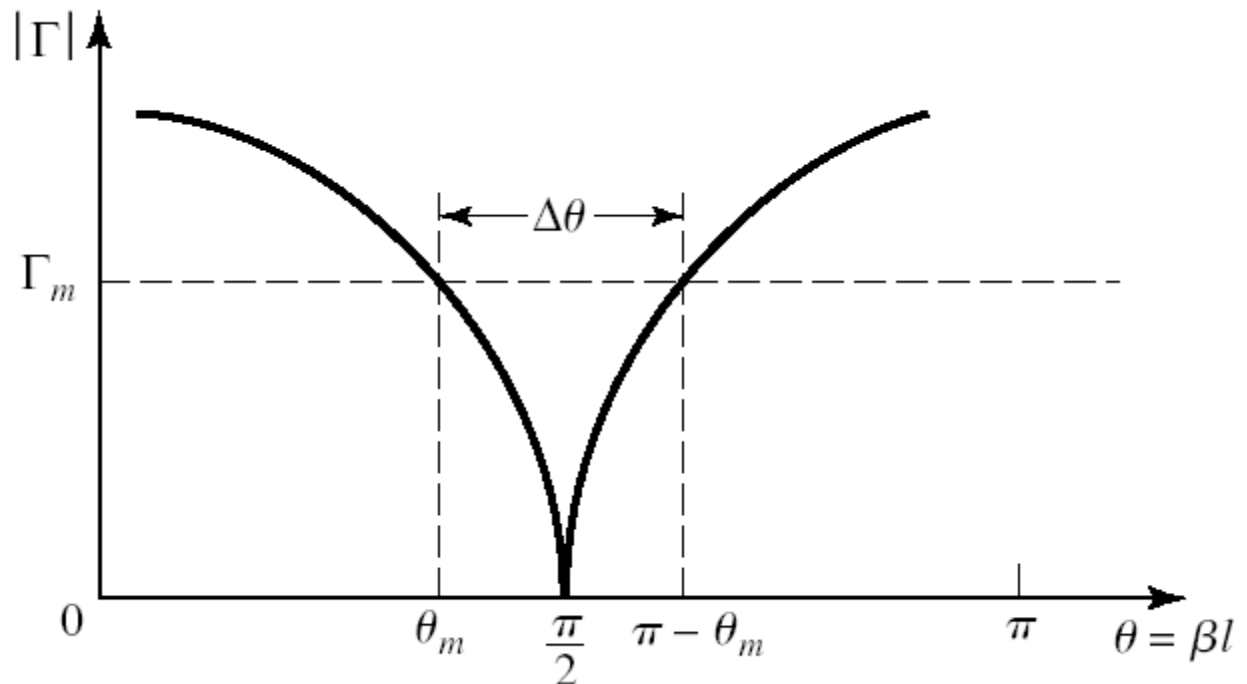
$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + t^2$$

Caracteristica de frecventa

- ne intereseaza frecventa in jurul frecventei la care facem adaptarea (banda ingusta)

$$f \approx f_0 \quad l \approx \frac{\lambda_0}{4} \quad \theta \approx \frac{\pi}{2} \quad \sec^2 \theta = 1 + \tan^2 \theta \gg 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos \theta|$$



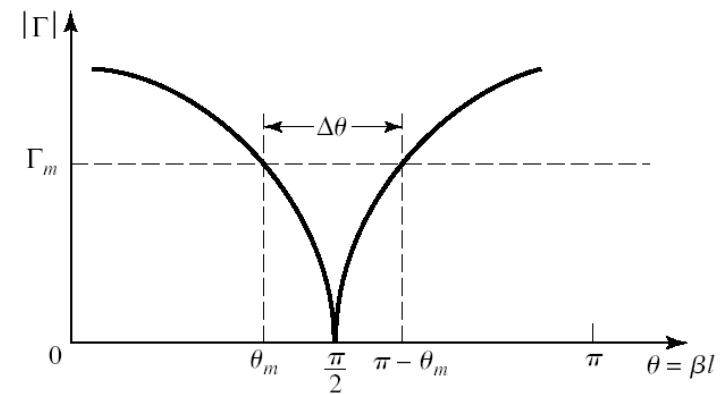
Caracteristica de frecventa

- Definim un maxim acceptat pentru coeficientul de reflexie Γ_m care va defini banda adaptarii, θ_m

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0} \sec \theta_m \right)^2,$$

$$\cos \theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2 \left(\frac{\pi}{2} - \theta_m \right)$$



- in linii TEM

$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2f_0}$$

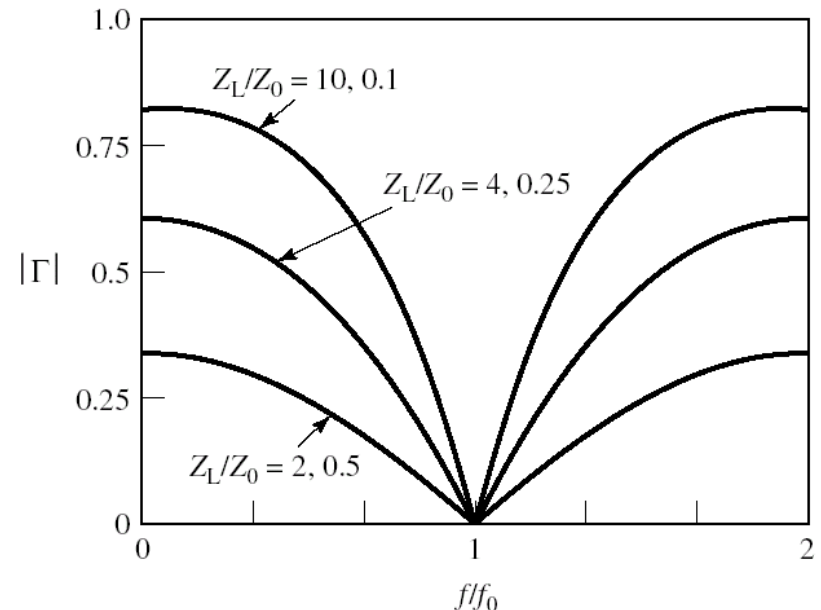
$$f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|} \right]$$

Caracteristica de frecventa

- Pentru linii non TEM constanta de propagare nu depinde liniar de frecventa, dar in practica influenta este minora in banda ingusta
- Sunt neglijate reactantele introduse de discontinuitati ($Z_0 \rightarrow Z_1$). Compensarea se face printr-o mica modificare a lungimii liniei
- Banda depinde de dezadaptarea initiala

cu cat dezadaptarea este mai mica
cu atat banda se obtine mai larga



Exemplu

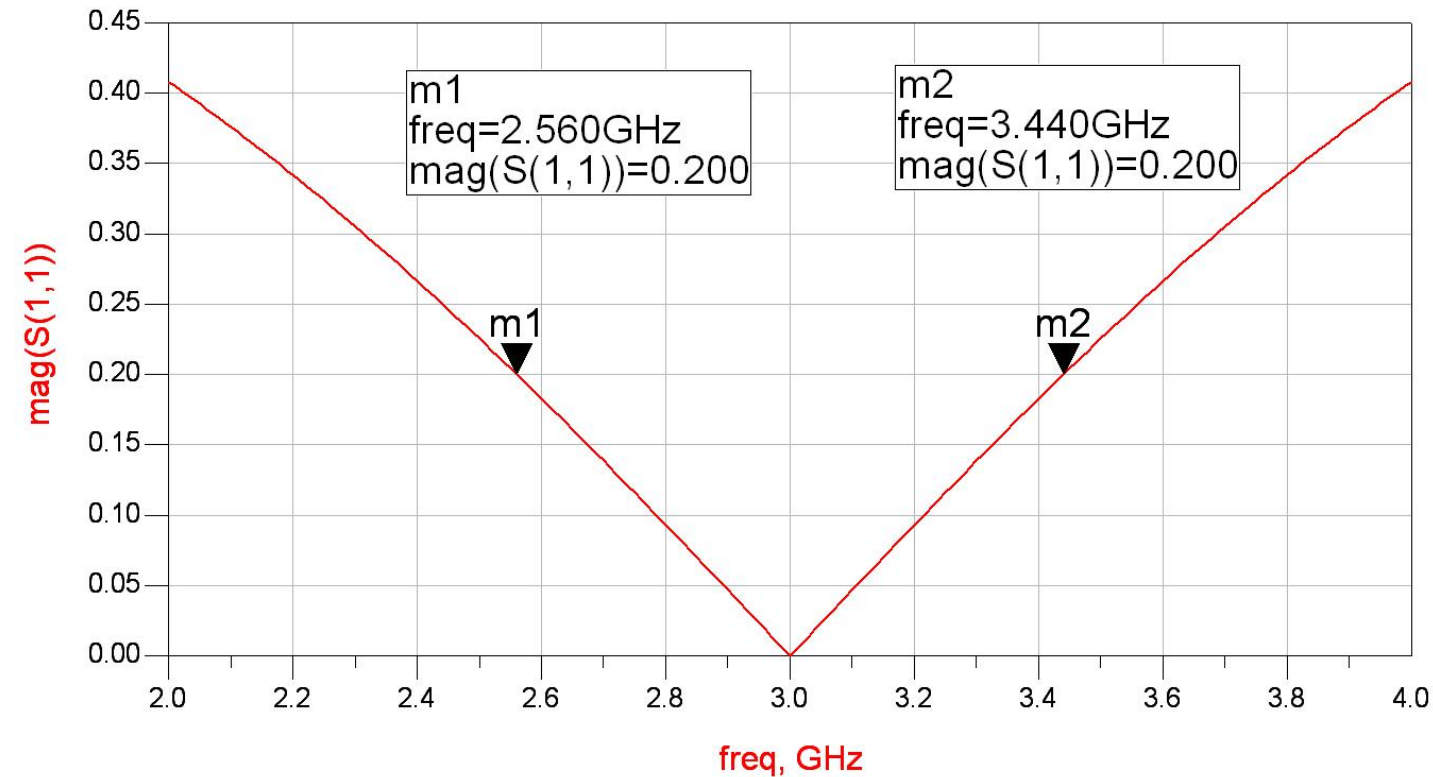
- Transformator de adaptare cu o singura sectiune ($\lambda/4$) pentru a adapta o sarcina de 10Ω la o linie de 50Ω la frecventa $f_0=3\text{GHz}$
 - banda pentru $\text{SWR}<1.5$

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\Omega, \quad \Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

$$\begin{aligned} \frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|} \right] \\ &= 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|} \right] \\ &= 0.29, \text{ or } 29\%. \end{aligned}$$

Simulare

■ simulare ADS

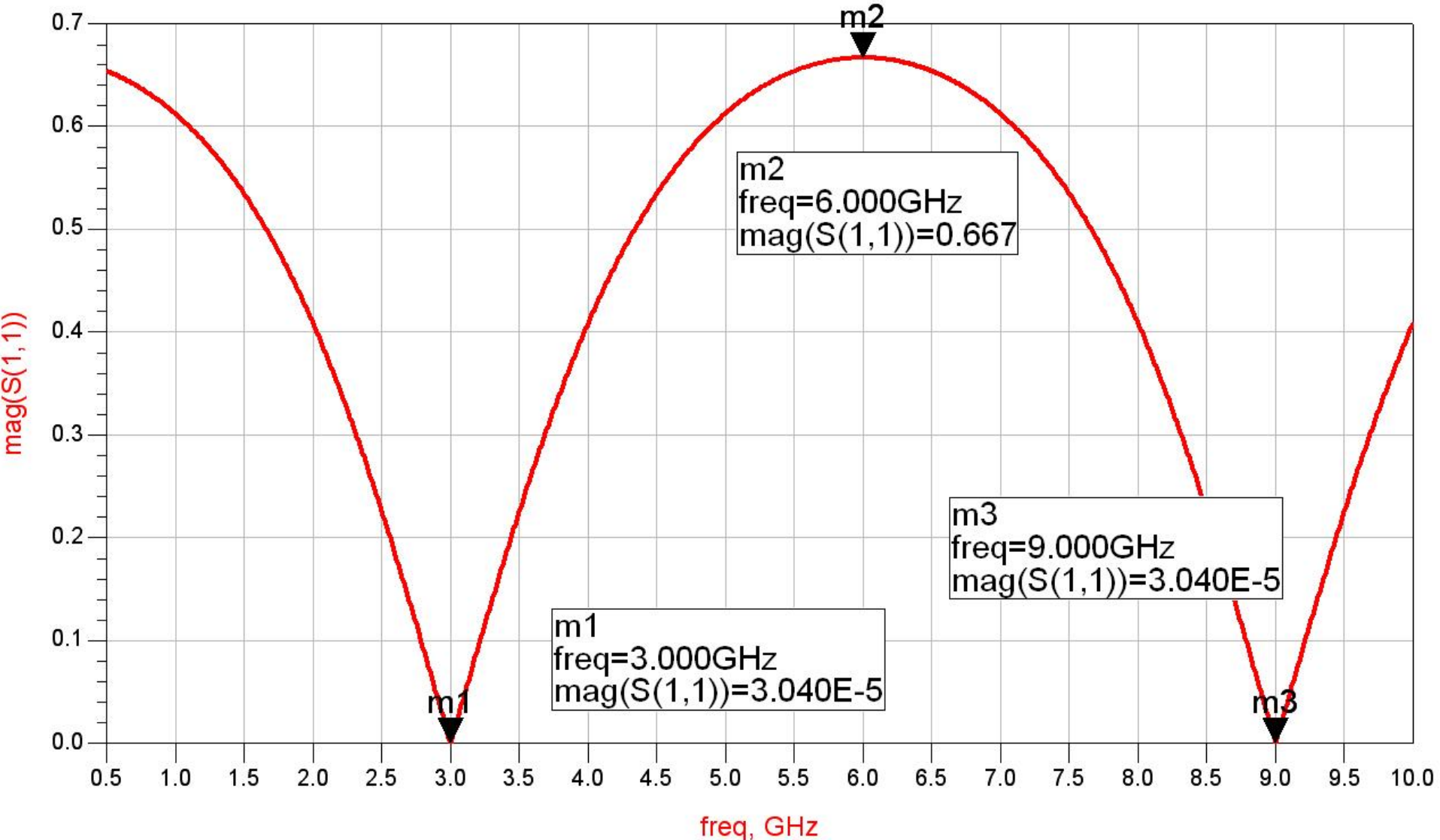


$$\Delta f = 0.88\text{GHz}$$

$$|\Gamma(3\text{GHz})| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

Simulare banda larga



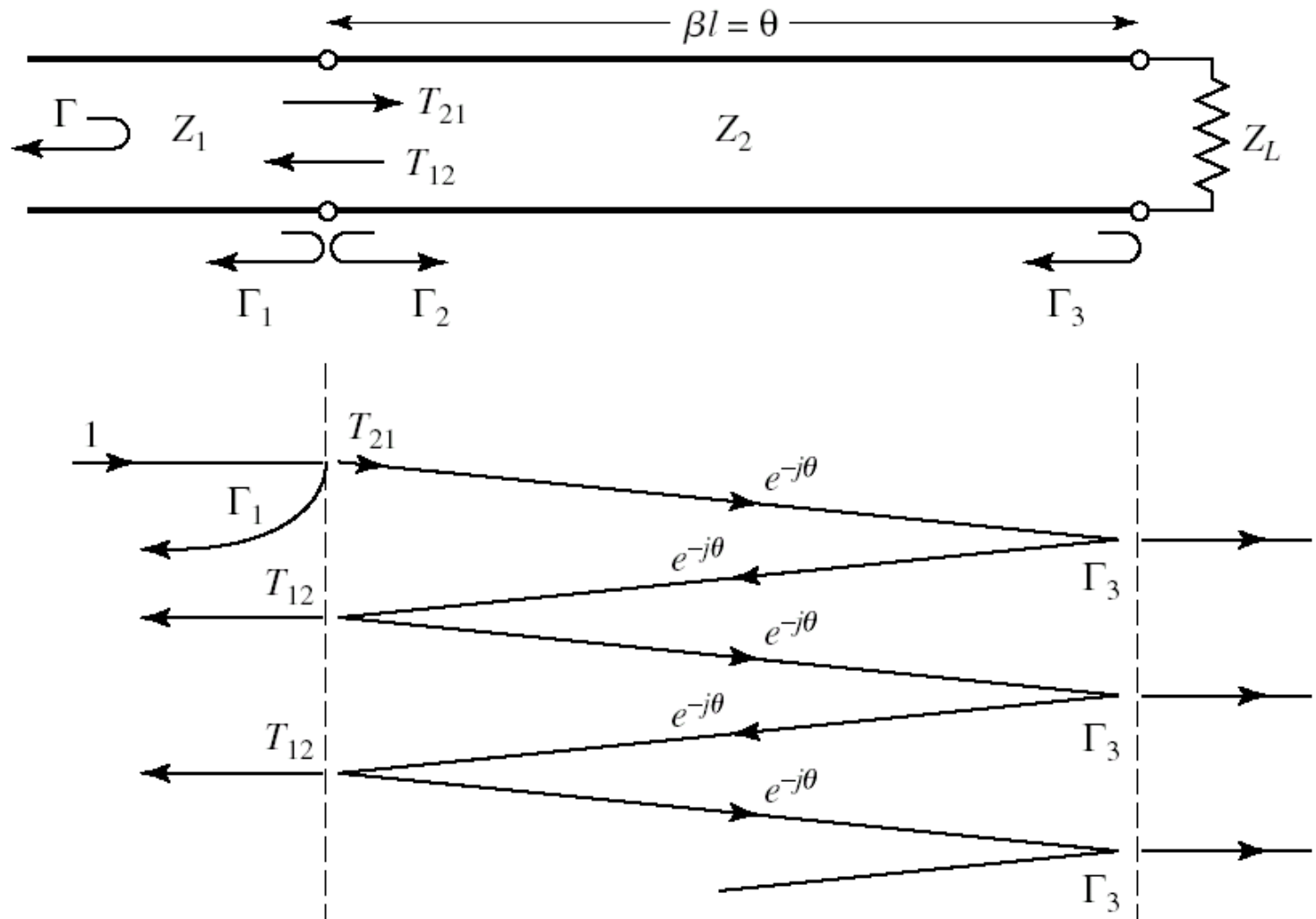
Adaptarea cu transformatoare de impedanta (Lab. 1)

Adaptarea de impedanță

Transformatoare de impedanta multisectiune

- Transformatorul in sfert de lungime de unda permite adaptarea oricarei impedante reale cu orice impedanta a fiderului (liniei).
- Daca banda necesara este mai mare decat cea oferita de transformatorul in sfert de lungime de unda se folosesc transformatoare multisectiune
 - caracteristica binomiala
 - tip Cebîșev

Teoria reflexiilor mici



Teoria reflexiilor mici

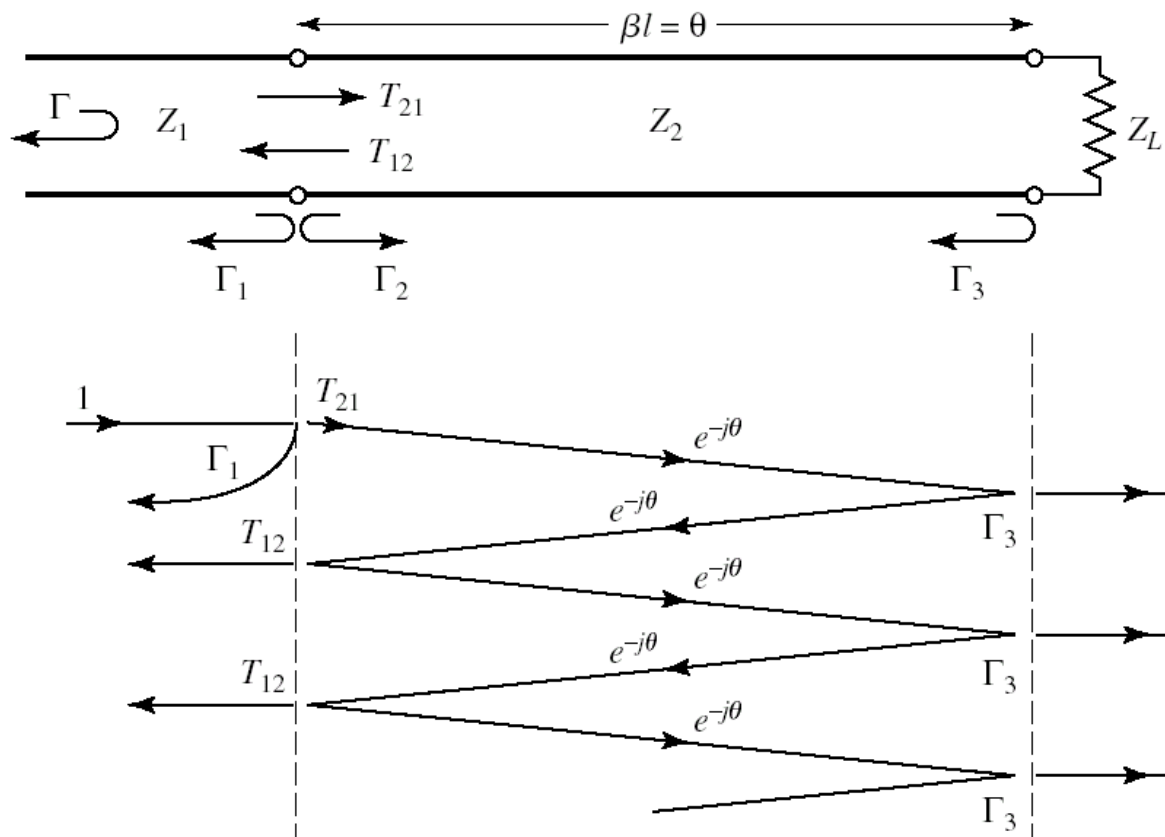
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$



$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \dots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

Teoria reflexiilor mici

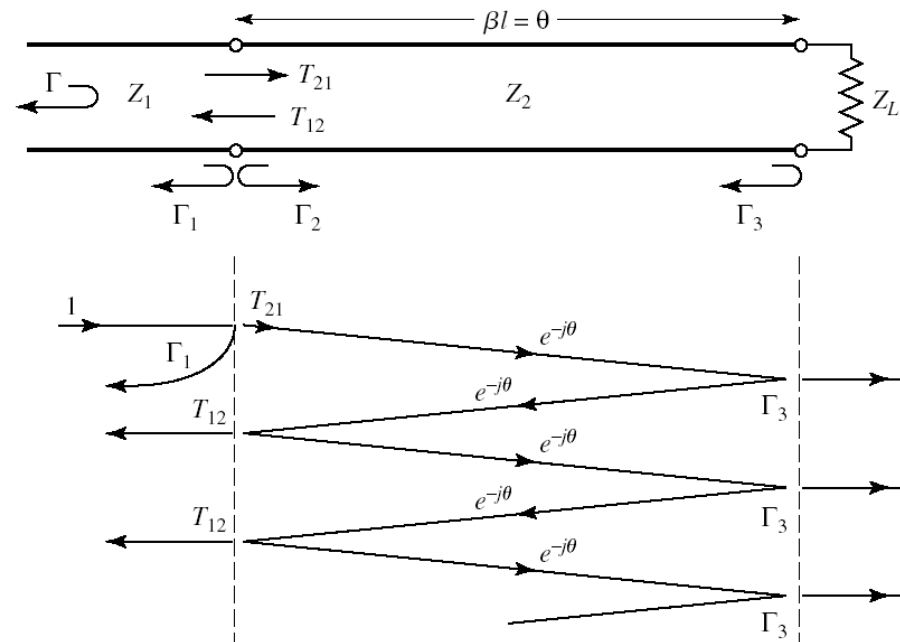
$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

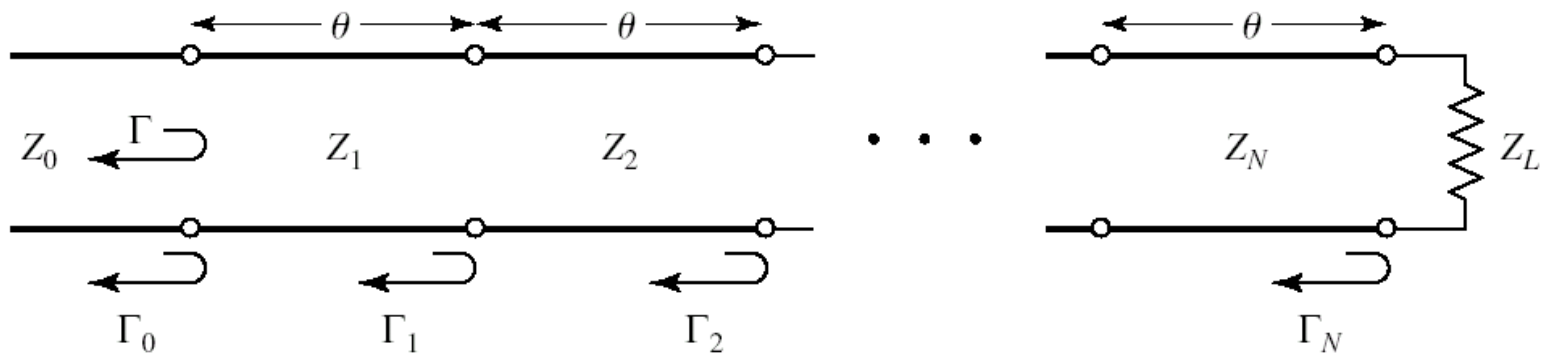
$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

- Dacă diferențele între $Z_1 \div Z_2$ și $Z_2 \div Z_L$ sunt mici putem aproxima:

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



Transformatoare cu mai multe sectiuni



- Presupunem ca toate impedantele **cresc sau descreasc uniform**

- Toti coeficientii de reflexie vor fi reali si de acelasi semn

- Anterior $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

Transformatoare cu mai multe sectiuni

- Realizez transformatorul **simetric**

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \dots$$

- Aceasta **nu** implica faptul ca impedantele sunt egale

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot [\Gamma_0 \cdot (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 \cdot (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \Gamma_2 \cdot (e^{j(N-4)\theta} + e^{-j(N-4)\theta}) + \dots]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:

$$\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$

Transformatoare cu mai multe sectiuni

- Coeficient de reflexie

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \dots + a_N \cdot x^N$$

- aleg coeficientii astfel incat sa obtin o variatie dorita (a polinomului)

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Raspunsul acestui transformator este de tip maxim plat in jurul frecventei de adaptare
- Pentru N sectiuni se anuleaza primele N-1 derivate ale functiei $|\Gamma(\theta)|$

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N$$

$$|\Gamma(\theta)| = |A| \cdot |e^{-j\theta}|^N \cdot |e^{j\theta} + e^{-j\theta}|^N = 2^N \cdot |A| \cdot |\cos \theta|^N$$

$$l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2} \quad \left| \Gamma\left(\frac{\pi}{2}\right) \right| = 0; \quad \frac{d^n}{d\theta^n} |\Gamma(\theta)| \Big|_{\theta=\frac{\pi}{2}} = 0 \quad n = \overline{1, N-1}$$

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- $A, \theta \rightarrow 0$, liniile de lungime 0, dispar

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- dezvoltarea binomului

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \dots + C_N^n \cdot x^n + \dots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!n!}$$

- Coeficientii de reflexie

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N \quad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

■ Proiectare, solutii aproximative

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}$$

$$\ln x \cong 2 \cdot \frac{x-1}{x+1} \quad x \cong 1$$

$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cdot C_N^n \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

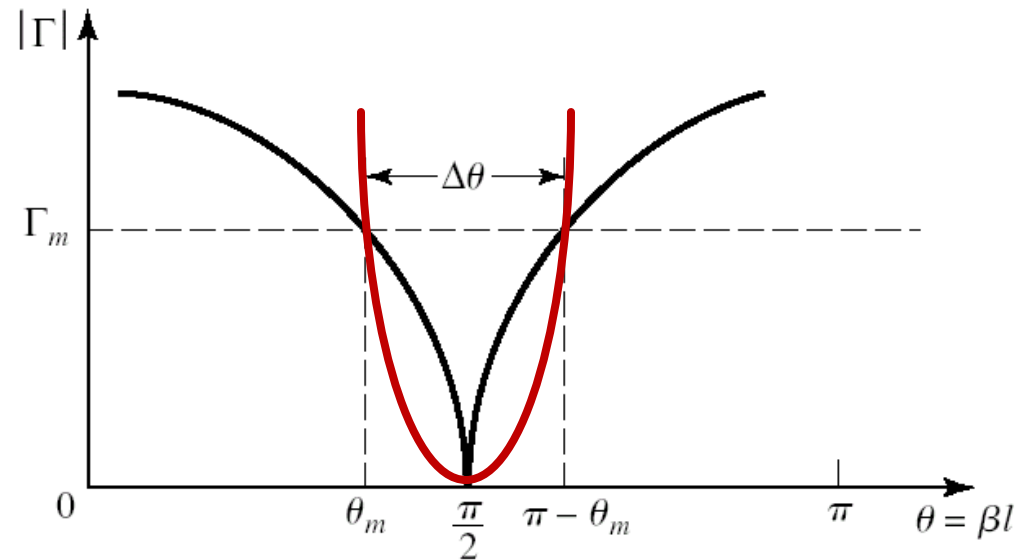
$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Banda, Γ_m maxim tolerat

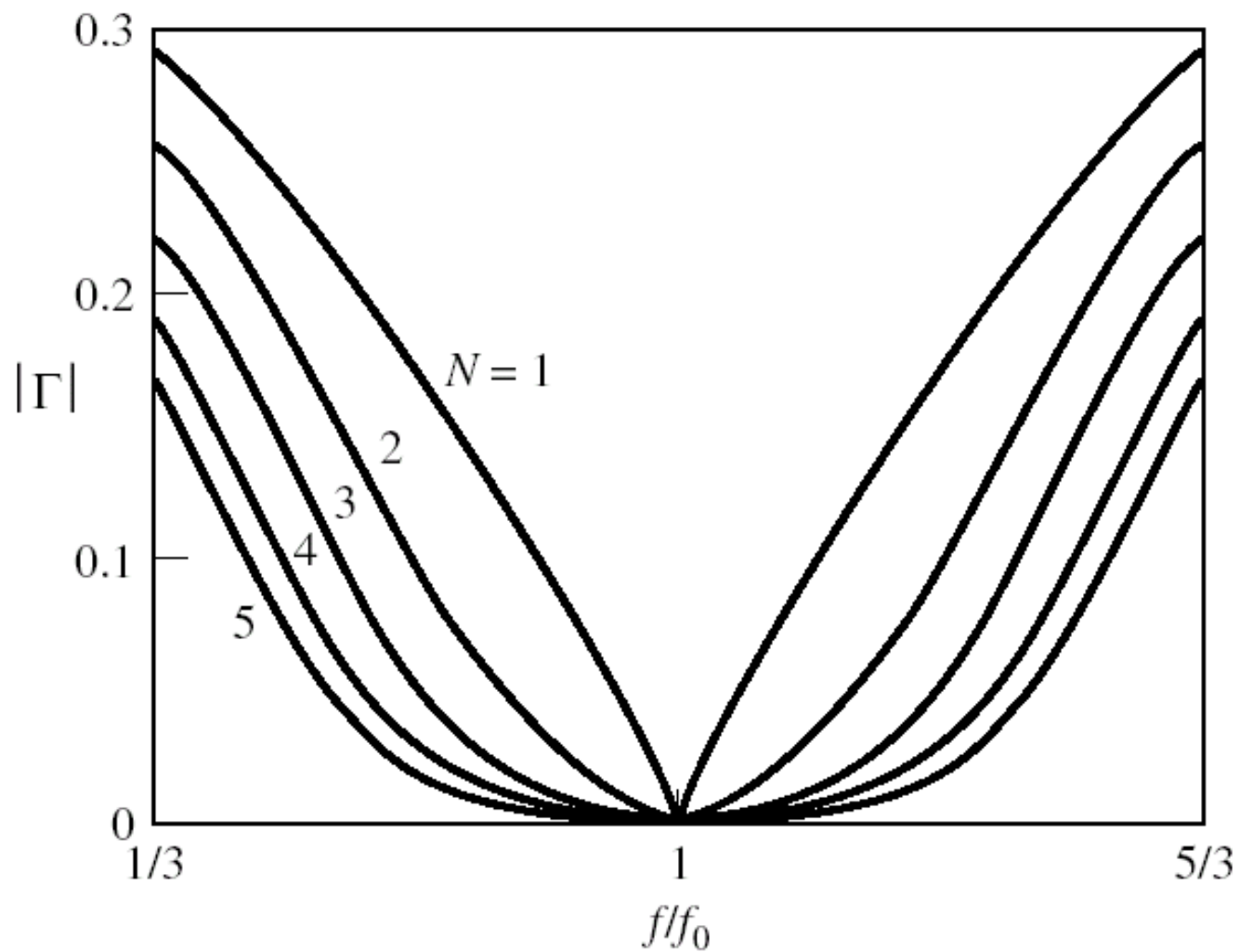
$$\Gamma_m = |\Gamma(\theta_m)| = 2^N \cdot |A| \cdot |\cos \theta_m|^N$$

$$\theta_m = \cos^{-1} \left[\frac{1}{2} \left(\frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$



$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[\frac{1}{2} \left(\frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$

Banda



Transformatoare cu mai multe sectiuni cu caracteristica binomiala, rezultate exacte

Z_L/Z_0	$N = 2$		$N = 3$			$N = 4$			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	$N = 5$					$N = 6$					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de 30Ω la o linie de 100Ω la frecventa $f_0=3\text{GHz}$, $\Gamma_m=0.1$

- $N = 3$

$$Z_L = 30\Omega \quad Z_0 = 100\Omega$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3! \cdot 0!} = 1 \quad C_3^1 = \frac{3!}{2! \cdot 1!} = 3 \quad C_3^2 = \frac{3!}{1! \cdot 2!} = 3$$

Exemplu

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03 \Omega$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77 \Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87 \Omega$$

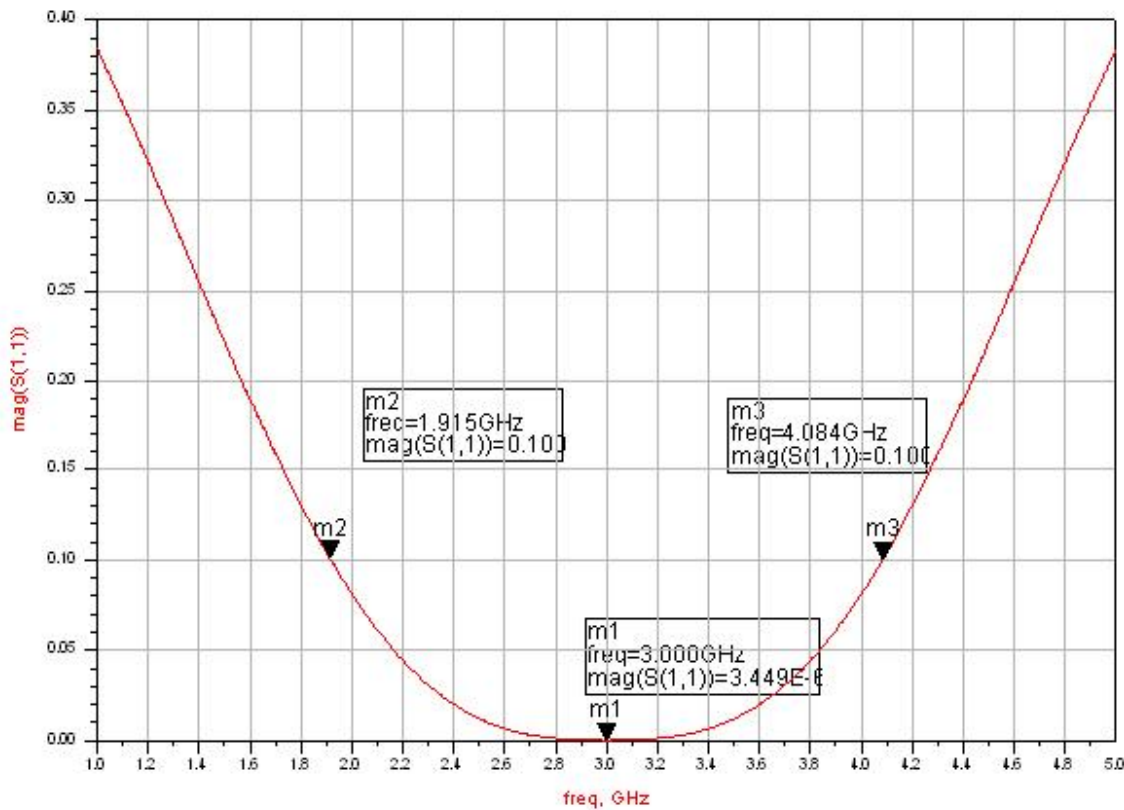
Exemplu

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \arccos \left[\frac{1}{2} \left(\frac{\Gamma_m}{|A|} \right)^{1/N} \right] = 2 - \frac{4}{\pi} \arccos \left[\frac{1}{2} \left(\frac{0.1}{0.07525} \right)^{1/3} \right] = 0.74$$

$$\Delta f = 2.22 \text{GHz}$$

Simulare

■ Similar Lab. 1



$$\Delta f = 2.169\text{GHz}$$

$$|\Gamma(3\text{GHz})| = 3.5 \cdot 10^{-6}$$

Transformatoare cu mai multe sectiuni de tip Cebîșev

- Raspunsul acestui transformator este de tip echiriplu in jurul frecventei de adaptare
- mareste banda in detrimentul riplului (variatiilor) in banda de adaptare
- Se egaleaza functia $\Gamma(\theta)$ cu un polinom Cebîșev

Polinoame Cebîşev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

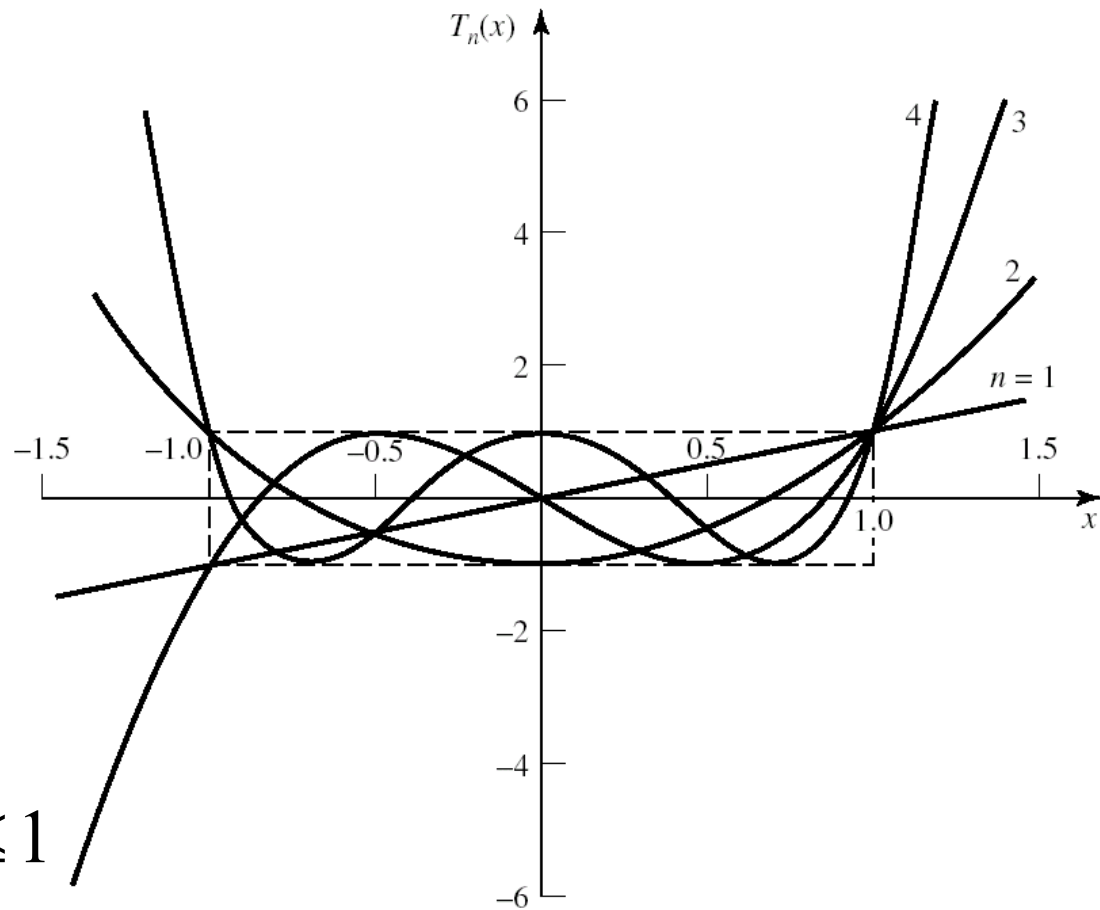
$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

■ Echiriplu

$$-1 \leq x \leq 1 \Rightarrow |T_n(x)| \leq 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$



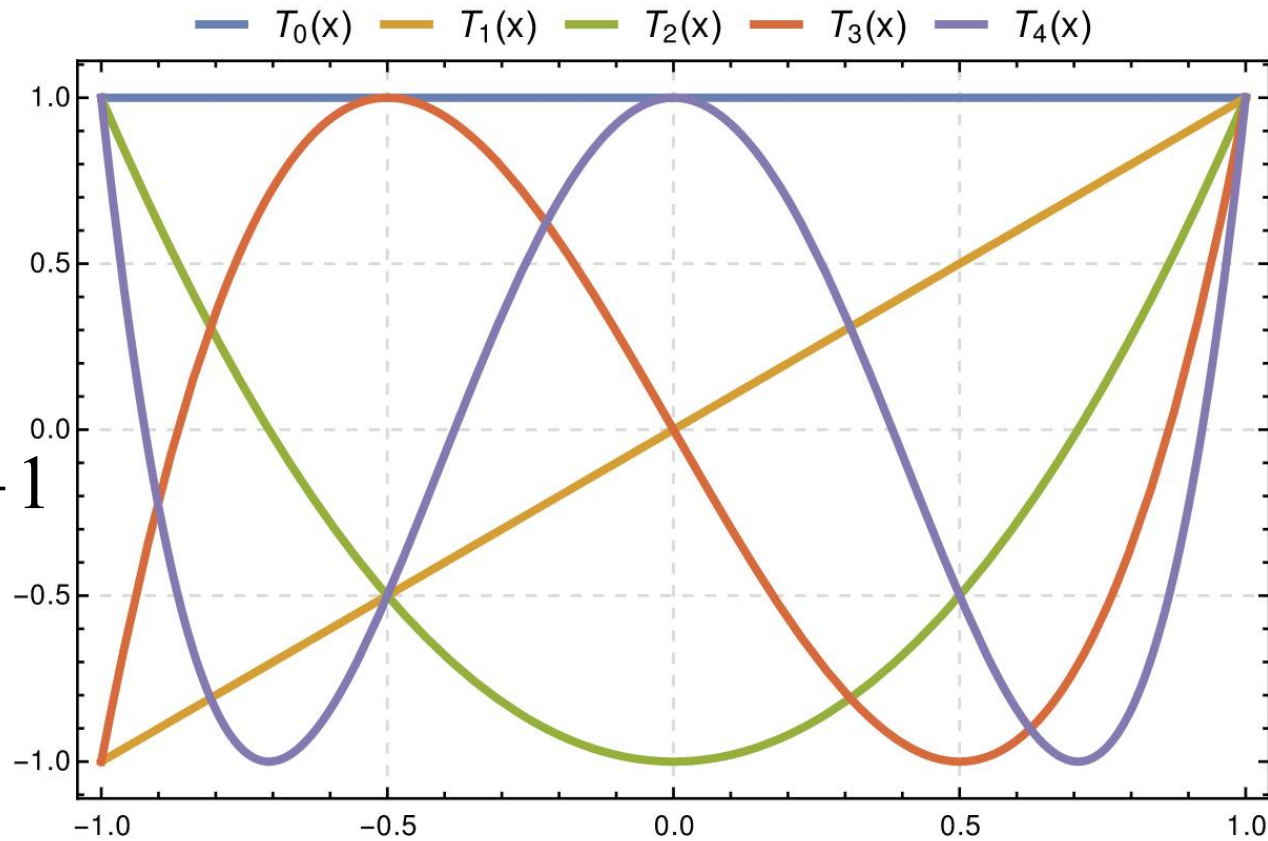
Polinoame Cebîşev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



■ Echiriplu

$$-1 \leq x \leq 1 \quad \Rightarrow \quad |T_n(x)| \leq 1$$

Polinoame Cebîșev

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \dots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\begin{aligned} & \dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par} \\ \text{ultimul termen:} & \dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar} \end{aligned}$$

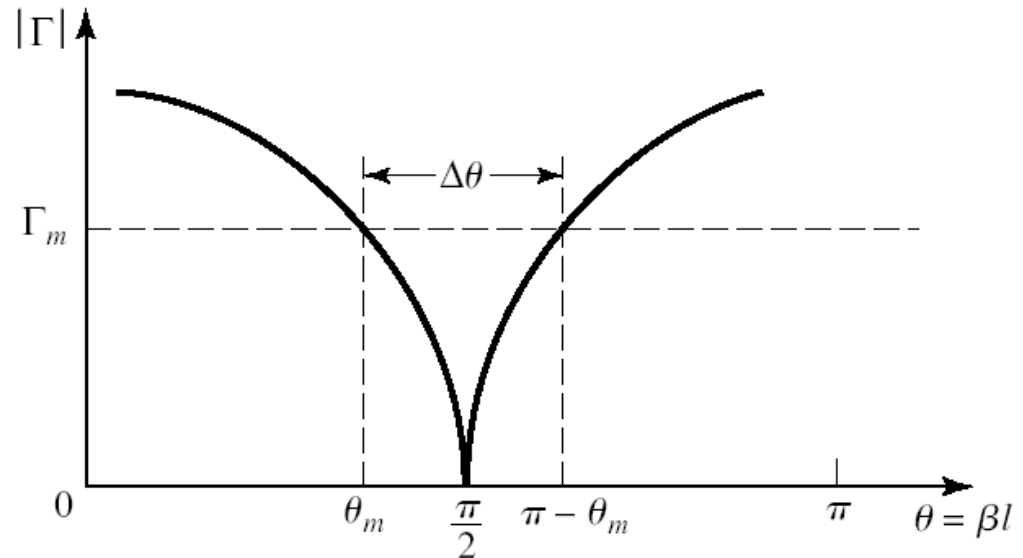
$$x = \cos \theta \quad |x| < 1$$

■ Se poate arata ca: $T_n(\cos \theta) = \cos(n\theta)$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \quad T_n(x) = \cosh(n \cosh^{-1}(x)) \quad |x| > 1$$

Transformatoare cu mai multe sectiuni de tip Cebîșev

- Schimbare de variabila
 - banda de interes
→ $[-1,1]$



$$\theta = \theta_m \rightarrow x = 1 \quad \theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos \theta}{\cos \theta_m}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$x = \sec \theta_m \cos \theta$$

Transformatoare cu mai multe sectiuni de tip Cebîșev

- Cautam coeficientii pentru a obtine un polinom Cebîșev

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:

$$\dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar}$$


$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec \theta_m \cos \theta)$$

Polinoame Cebîşev

$$\cancel{(\cos \theta)^k} \Leftrightarrow \cos k\theta$$

$$T_1(x) = x$$

$$T_1(\sec \theta_m \cos \theta) = \sec \theta_m \cos \theta$$

$$T_2(x) = 2x^2 - 1$$

$$T_2(\sec \theta_m \cos \theta) = 2 \sec^2 \theta_m \cos^2 \theta - 1$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$$

$$T_2(\sec \theta_m \cos \theta) = \sec^2 \theta_m (1 + \cos 2\theta) - 1$$

$$T_3(x) = 4x^3 - 3x \quad T_3(\sec \theta_m \cos \theta) = 4 \sec^3 \theta_m \cos^3 \theta - 3 \sec \theta_m \cos \theta$$

$$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$$

$$\cos 3\theta = (2 \cos^2 \theta - 1) \cos \theta - 2(1 - \cos^2 \theta) \cos \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$T_3(\sec \theta_m \cos \theta) = \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3 \sec \theta_m \cos \theta$$

Transformatoare cu mai multe sectiuni de tip Cebîșev

$$T_1(\sec \theta_m \cos \theta) = \sec \theta_m \cos \theta$$

$$T_2(\sec \theta_m \cos \theta) = \sec^2 \theta_m (1 + \cos 2\theta) - 1$$

$$T_3(\sec \theta_m \cos \theta) = \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3 \sec \theta_m \cos \theta$$

$$T_4(\sec \theta_m \cos \theta) = \sec^4 \theta_m (\cos 4\theta + 4 \cos 2\theta + 3) - 4 \sec^2 \theta_m (\cos 2\theta + 1) + 1$$

- Cautam coeficientii pentru a obtine un polinom Cebîșev

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec \theta_m \cos \theta)$$

$$\begin{aligned} \text{ultimul termen:} \quad & \dots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par} \\ & \dots \Gamma_{(N-1)/2} \cdot \cos \theta \quad n \text{ impar} \end{aligned}$$

Transformatoare cu mai multe sectiuni de tip Cebîșev

- $A, \theta \rightarrow 0$, liniile de lungime 0, dispar

$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = A \cdot T_N(\sec \theta_m) \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)}$$

$$\Gamma_m = |A|$$

$$T_N(\sec \theta_m) = \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \cong \frac{1}{2\Gamma_m} \left| \ln \frac{Z_L}{Z_0} \right|$$

$$T_n(x) = \cosh(n \cosh^{-1}(x))$$

$$\sec \theta_m = \cosh \left[\frac{1}{N} \cosh^{-1} \left(\frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \right) \right] \cong \cosh \left[\frac{1}{N} \cosh^{-1} \left(\left| \frac{\ln(Z_L/Z_0)}{2\Gamma_m} \right| \right) \right]$$

- se poate calcula θ_m pentru Γ_m maxim tolerat (riplu) deci banda

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

Transformatoare cu mai multe sectiuni de tip Cebîșev

- Proiectare, solutii aproximative

$$\Gamma_m = |A|$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)}$$

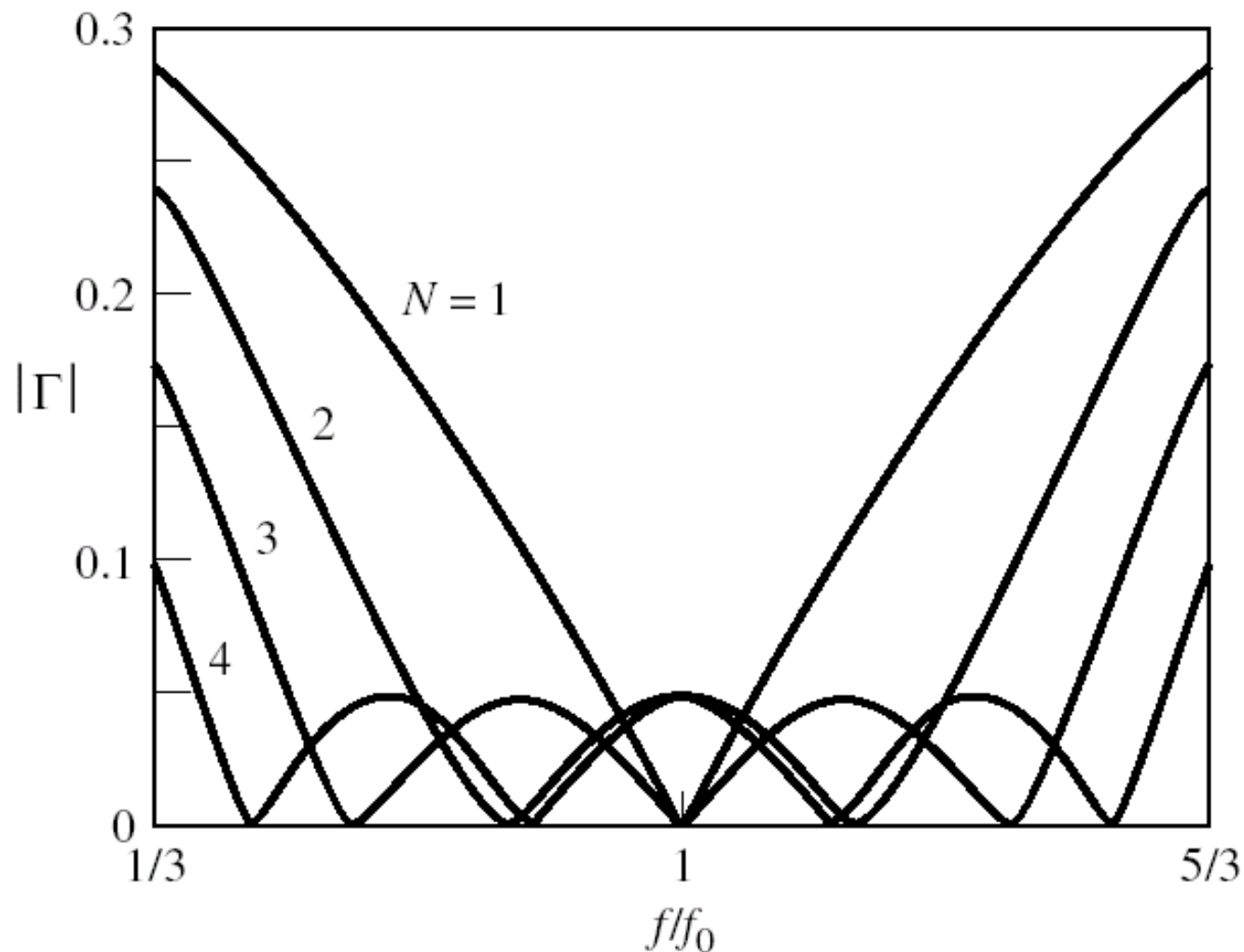
- Semnul lui A depinde de $Z_L \lessgtr Z_0$
- Se calculeaza $\sec \theta_m$
- Se reprezinta polinomul Cebîșev de ordin ales si se identifica coeficientii pentru $\cos k\theta$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n$$

$$\ln Z_{n+1} \cong \ln Z_n + 2 \cdot \Gamma_n$$

Transformatoare cu mai multe sectiuni de tip Cebîșev



Transformatoare cu mai multe sectiuni de tip Cebîșev, rezultate exacte

Z_L/Z_0	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

Z_L/Z_0	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de 30Ω la o linie de 100Ω la frecventa $f_0=3\text{GHz}$, $\Gamma_m=0.1$
 - $N = 3$ $Z_L = 30\Omega$ $Z_0 = 100\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta} [\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = Ae^{-j3\theta} T_3(\sec \theta_m \cos \theta)$$

$$|A| = \Gamma_m = 0.1 \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad Z_L < Z_0 \rightarrow A < 0 \quad A = -0.1$$

$$\sec \theta_m = \cosh \left[\frac{1}{N} \cdot \cosh^{-1} \left(\left| \frac{\ln Z_L / Z_0}{2\Gamma_m} \right| \right) \right] = \cosh \left[\frac{1}{3} \cdot \cosh^{-1} \left(\left| \frac{\ln(30/100)}{2 \cdot 0.1} \right| \right) \right] = 1.362$$

$$\theta_m = \arccos \left(\frac{1}{\sec \theta_m} \right) = 0.746 \text{ rad} = 42.76^\circ$$

Exemplu

$$2[\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = A \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3A \sec \theta_m \cos \theta$$

$\cos 3\theta$

$$2\Gamma_0 = A \sec^3 \theta_m$$

$$\Gamma_0 = -0.1263$$

$\cos \theta$

$$2\Gamma_1 = 3A(\sec^3 \theta_m - \sec \theta_m)$$

$$\Gamma_1 = -0.1747$$

simetrie: $\Gamma_3 = \Gamma_0; \quad \Gamma_2 = \Gamma_1$

Exemplu

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353 \quad \Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega \quad \Gamma_1 = -0.1747$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

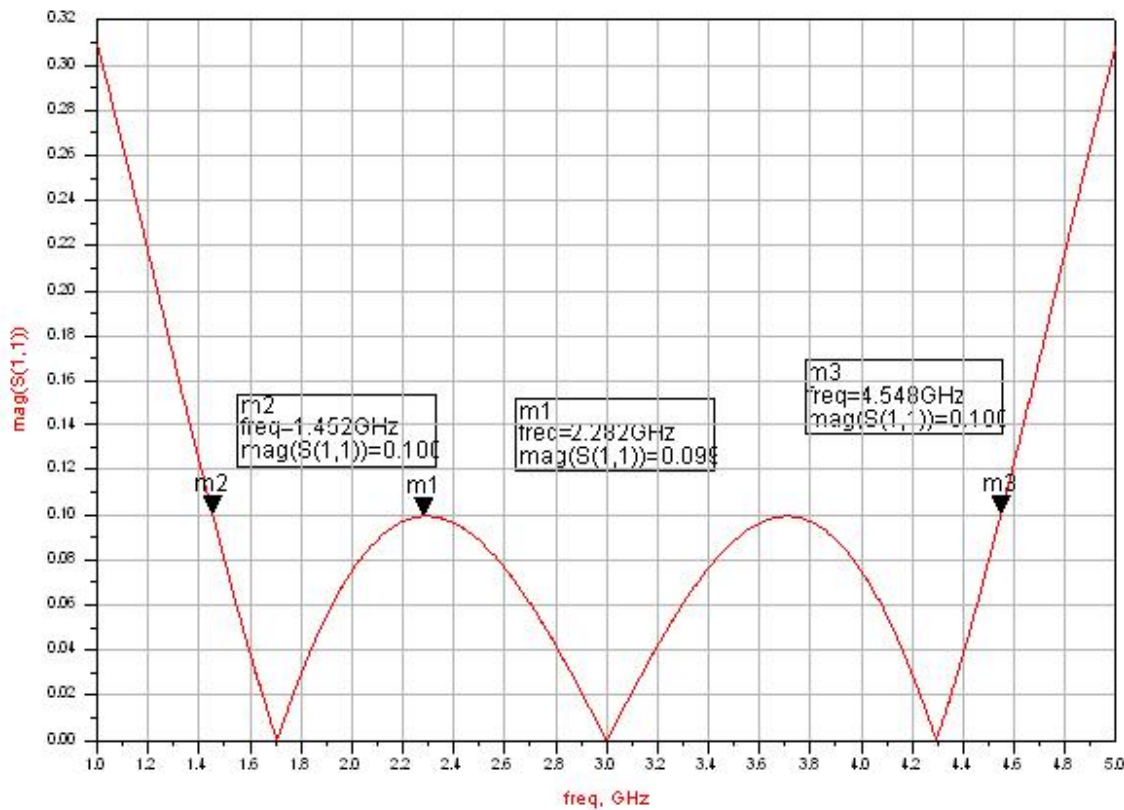
Exemplu

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76^\circ}{180^\circ} = 1.045$$

$$\Delta f = 3.15 \text{ GHz}$$

Simulare

■ Similar Lab. 1



$$\Delta f = 3.096 \text{ GHz}$$

$$|\Gamma(3 \text{ GHz})| = 4.17 \cdot 10^{-5}$$

$$|\Gamma(2.282 \text{ GHz})| = 0.09925$$

Solutii exacte

- G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House Books, Dedham, Mass. 1980

Puncte suplimentare

Tema

Tema

- Scrieti pe o foaie de hartie (si scanati/foto) relatiile pentru:
 - determinarea conditiei de maxim de putere utilizand derivate partiale (impedante complexe)

$$\frac{\partial P_L}{\partial R_L} = 0 \quad \frac{\partial P_L}{\partial X_L} = 0$$

- determinarea expresiei pentru polinomul Cebîșev de ordinul 4

$$T_4(\sec \theta_m \cos \theta) = \sec^4 \theta_m (\cos 4\theta + 4 \cos 2\theta + 3) - 4 \sec^2 \theta_m (\cos 2\theta + 1) + 1$$

Contact

- Laboratorul de microunde si optoelectronica
- <https://rf-opto.etti.tuiasi.ro>
- rdamian@etti.tuiasi.ro